Structures, waves and turbulence in the heliosphere

- Fluctuations - scales and parameters
- Magnetoacoustic and Alfvénic fluctuations
- Turbulence spectra and radial evolution
- Ideal MHD invariants and dissipation
- Cross-helicity, anisotropy, compressibility
- Scaling and intermittency
- Shock waves and discontinuities
- Plasma waves
The Sun’s open magnetic field lines

MHD model field during Ulysses crossing of ecliptic plane in early 1995
Waves and turbulence on open fields

- Photospheric flux tubes are shaken by an observed spectrum of horizontal motions.
- Alfvén waves propagate along the field, and may partly reflect back down.
- Nonlinear couplings force a (perpendicular?) cascade, terminated by damping.

Solar wind stream structure and heliospheric current sheet

Parker, 1963

Alfven, 1977
Stream interaction region

Dynamic processes in interplanetary space

- Wave amplitude steepening ($n \sim r^{-2}$)
- Compression and rarefaction
- Velocity shear
- Nonlinearity by advection \( (\mathbf{V} \cdot \nabla) \mathbf{V} \)
- Shock formation (co-rotating)

Schwenn, 1990
## Spatial and temporal scales

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Frequency ($s^{-1}$)</th>
<th>Period (day)</th>
<th>Speed (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar rotation:</td>
<td>4.6 $10^{-7}$</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>Solar wind expansion:</td>
<td>5 - 2 $10^{-6}$</td>
<td>2 - 6</td>
<td>800 - 250</td>
</tr>
<tr>
<td>Alfvén waves:</td>
<td>3 $10^{-4}$</td>
<td>1/24</td>
<td>50 (1AU)</td>
</tr>
<tr>
<td>Ion-cyclotron waves:</td>
<td>1 - 0.1</td>
<td>1 (s)</td>
<td>($V_A$) 50</td>
</tr>
</tbody>
</table>

**Turbulent cascade:**

generation + transport $\rightarrow$ inertial range $\rightarrow$ kinetic range + dissipation
Fluctuations

Typical day in April 1995 of Ulysses plasma and field observations in the polar (42° north) heliosphere at 1.4 AU

- Sharp changes in field direction
- Large Component variations
- Weakly compressive fluctuations

Horbury & Tsurutani, 2001
Phase velocities of MHD modes

\[ \omega^4 - \omega^2 (kc_{ms})^2 + (kc_s)^2 (k \cdot V_A)^2 = 0 \]

\[ \omega = k \cdot V_A \]
Weak turbulence, superposition of magnetohydrodynamic waves

- Magnetosonic waves
  - compressible
  - parallel slow and fast
  - perpendicular fast

\[ C_{ms} = (c_s^2 + V_A^2)^{-1/2} \]

- Alfvén wave
  - incompressible
  - parallel and oblique

\[ V_A = B/(4\pi \rho)^{1/2} \]

Broad band in \( \mathbf{k} \) and random phases
Alfvénic fluctuations (Helios)

\[ \delta V = \pm \delta V_A \]
Alfvénic fluctuations (Ulysses)

Elsässer variables:
\[ Z^\pm = V \pm V_A \]

Turbulence energy:
\[ e^\pm = \frac{1}{2} (Z^\pm)^2 \]

Cross helicity:
\[ \sigma_c = (e^+ - e^-)/(e^+ + e^-) \]
Alfvénic fluctuations

Ulysses observed many such waves (4-5 per hour) in fast wind over the poles:

- Arc-polarized waves
- Phase-steepened

Rotational discontinuity:

\[ \Delta \mathbf{V} = \pm \Delta \mathbf{V}_A \]

Finite jumps in velocities over gyrokinetic scales

Tsurutani et al., 1997
Arc-polarized Alfvén waves

Slowly rotating Alfvén wave lasts about 15 minutes

Rotational discontinuity RD lasts only 3 minutes

Tsurutani et al., 1997
Alfvén waves and solar wind streams in the ecliptic plane

- High Alfvén wave flux in fast streams
- Developed isotropic turbulence in slow streams

Tu et al., GRL, 17, 283, 1990
Alfvén waves in polar solar wind

Elsässer variables: \( Z^\pm = V \pm V_A \)

Turbulence energy: \( e^\pm = \frac{1}{2} (Z^\pm)^2 \)

Elsässer ratio: \( r_e = e^-/e^+ \)

Radial variation of \( e^\pm(r) \); wave amplitude at 1-h period is not sufficient to drive fast wind!

Average values over 0.1 AU wide intervals of hourly variances of \( Z^\pm \)

Bavassano et al., J GR, 105, 15959, 2001
Anisotropy and dimension

Correlations:
Alfvén waves and 2-D turbulence

Matthaeus et al., J. Geophys. Res., 95, 20673, 1990
Two-component model

- Alfvén waves parallel to the mean field
- 2-dimensional turbulence perpendicular
- Convected structures (discontinuities) and shocks

Flux tube angular scale: 2° - 4°; supergranule: 20-30 Mm
Compressive fluctuations

Colour coding:
Correlation coefficient (per solar rotation) between total plasma pressure $p_t$ and density $n$, and kinetic (thermal) pressure $p_k$ and magnetic pressure $p_m$, indicating magnetoacoustic slow mode type fluctuations.

Left scale:
Time, radial distance, and heliographic latitude of Ulysses.

Bavassano et al., Ann. Geophysicae., 2004
Compressive fluctuations in the solar wind

Marsch and Tu, JGR, 95, 8211, 1990
Kolmogorov-type turbulence
# Solar wind turbulence

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coronal Hole (open)</th>
<th>Current sheet (closed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alfvén waves:</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Density fluctuations:</td>
<td>weak (&lt;3%)</td>
<td>intense (&gt;10%)</td>
</tr>
<tr>
<td>Magnetic/kinetic turbulent energy:</td>
<td>≅ 1</td>
<td>&gt; 1</td>
</tr>
<tr>
<td>Spectral slope:</td>
<td>flat (-1)</td>
<td>steep (-5/3)</td>
</tr>
<tr>
<td>Wind speed:</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>$T_p (T_e)$:</td>
<td>high (low)</td>
<td>low (high)</td>
</tr>
<tr>
<td>Wave heating:</td>
<td>strong</td>
<td>weak</td>
</tr>
</tbody>
</table>
Magnetic field power spectrum

- Power laws with index of about -1, -5/3 and -3
- Abrupt decline at $f_c$ indicates cyclotron absorption
- Steep spectrum at high frequencies above 2 Hz is mainly due to whistler waves

Denskat et al., JGR 54, 60, 1983
Integral invariants of ideal MHD

\[ E = \frac{1}{2} \int d^3x (V^2 + V_A^2) \] Energy

\[ H_c = \int d^3x (V \cdot V_A) \] Helicity

\[ H_m = \int d^3x (A \cdot B) \] Magnetic helicity

\[ B = \nabla \times A \]

Elsässer variables:

\[ Z^\pm = V \pm V_A \]

\[ E^\pm = \frac{1}{2} \int d^3x (Z^\pm)^2 = \int d^3x e^{\pm}(x) \]
Correlation length of turbulence

\[ L_c = V_{SW} \tau_c \]

Correlation function:

\[ C_{AA'}(x,t,x',t') = \langle A(x,t) A(x',t') \rangle \]

for any field \( A(x,t) \).

If stationarity and homogeneity, then

\[ \tau = t - t', \quad r = x - x' \]

\[ C_{AA'}(x,t,x',t') = C_{AA'}(r, \tau) \]
Turbulence in the heliosphere

Questions and problems:

• Nature and origin of the fluctuations
• Distribution and spectral transfer of turbulent energy
• Spatial evolution with heliocentric distance
• Intermittency and microphysics of dissipation

Alfvénic correlations: Alfvénicity (cross helicity)

\[ \sigma_c = \frac{(e^+ - e^-)}{(e^+ + e^-)} = 2 \langle \delta V \cdot \delta V_A \rangle / \left( \langle (\delta V)^2 \rangle + \langle (\delta V_A)^2 \rangle \right) \]

Magnetic versus kinetic energy: Alfvén ratio

\[ r_A = \frac{e_v}{e_B} = \frac{\langle (\delta V)^2 \rangle}{\langle (\delta V_A)^2 \rangle} \]

Scaling, non-linear couplings and cascading?
Evolution of cross helicity

\[ \sigma_c = 2 \langle \delta \mathbf{V} \cdot \delta \mathbf{V}_A \rangle / (\delta \mathbf{V}^2 + \delta \mathbf{V}_A^2) \]

= \((e^+ - e^-)/(e^+ + e^-)\)

Alfvénic correlations decay radially!

Roberts et al., J. Geophys. Res. 92, 12023, 1987
Alfvén ratio

\[ r_A = \frac{e_V}{e_B} \]

Marsch and Tu, J. Geophys. Res., 95, 8211, 1990
Spectral indices and spatial evolution of turbulence

- Spectra steepen!
- $e^+ >> e^-$, Alfvén waves dominate!

Marsch and Tu, JGR, 95, 8211, 1990
Turbulence spectrum:
\[ e^{\pm}(f) = \frac{1}{2} (\delta Z^{\pm})^2 \sim \left(\frac{f}{f_0}\right)^{-\alpha} \]
Spectral evolution and turbulent cascade: slope steepening

Increasing time and distance
Radial variation of spectral features

- Turbulence intensity declines with solar distance
- Wave amplitudes are consistent between Helios and Ulysses in fast streams from coronal holes
- Variation of spectral breakpoint (decreases) as measured by various S/C
- Slower radial evolution of spectra over the poles

Horbury & Tsurutani, 2001
Spectral evolution of Alfvénic fluctuations

- Steepening by cascading
- Ion heating by wave sweeping
- Dissipation by wave absorption

Tu and Marsch, J. Geophys. Res., 100, 12323, 1995
## Kolmogorov phenomenology for isotropic homogeneous turbulence

**Energy cascade:**

Turbulent energy (per unit mass density), $e_l \approx (\delta Z)^2$, at scale $l$ is transported by a hierarchy of turbulent eddies of ever decreasing sizes to the dissipation range at scale $l_D$.

- **energy transfer rate:** $\varepsilon_l \sim (\delta Z)^2/\tau$
- **turnover time:** $\tau \sim l/\delta Z$
- **wavenumber:** $k \sim 1/l$
- **energy spectrum:** $E_k k \sim (\delta Z)^2$

$$\varepsilon_l \sim \delta Z/l (\delta Z)^2 \sim E_k^{3/2} k^{5/2}$$

**Scale invariance:** $\varepsilon_l = \varepsilon$ (dissipation rate) --> $E_k \sim k^{-5/3}$
Spectral properties of 3-D magnetohydrodynamic turbulence

Direct numerical simulation with a spectral code with \(512^3\) modes

Compensated normalized spectrum shows Kolmogorov scaling and sheet-like dissipative structures

\[ E_k \sim \varepsilon^{2/3} k^{-5/3} \quad \text{Kolmogorov, 1941} \]

\[ E_k \sim (\varepsilon v_A)^{1/2} k^{-3/2} \quad \text{Kraichnan, 1965} \]

MHD turbulence dissipation through absorption of plasma waves

- Viscous and Ohmic dissipation in collisionless plasma (coronal holes and fast solar wind) is not important.

- Waves become dispersive (at high frequencies beyond MHD) in the multi-fluid or kinetic regime.

- Question: KAW or Alfvén-cyclotron dissipation?

- Turbulence dissipation involves absorption (or emission by instability) of kinetic plasma waves!

- Cascading and spectral transfer of wave and turbulence energy is not well understood in the dispersive dissipation domain!
Anisotropic MHD cascade

- Simulations and analytic models predict cascade from small to large $k_\perp$, leaving $k_\parallel$ unchanged.

- **Critical balance** assumes $\omega_A = k_\parallel V_A \approx \omega_{NL} = k_\perp \delta V$ (Goldreich and Sridar, ApJ. 1995, 1997)

- **Kinetic Alfven wave** (KAW) with large $k_\perp$ does not necessarily have a high $\omega_A$.

- In a low-beta plasma, KAWs are Landau-damped, heating **electrons** preferentially!

Isocontours of model spectrum $E_k$
Cyclotron wave generation

**Base generation** by, e.g., “microflare” reconnection in the lanes that border convection cells (see Axford and McKenzie, 1997).

**Secondary generation** by low-frequency Alfven waves being converted by parallel cascading into cyclotron waves gradually in the corona.
**Structure function and scaling**

_Burlaga, JGR, 96, 5847, 1991_

\[ S^p (\tau) = < |V(\tau) - V(0)|^p > = \tau^{s(p)} \]

**Scaling exponent**

\[ s(p) = 1 - \ln \left[ \frac{P^p}{3} + \frac{(1-P)^p}{3} \right] \]

_P-model of fractal cascade; P = 1/2 no intermittency_
Radial evolution of intermittency

Helios, fast solar wind: $B_x$ radial component of magnetic field, $B_y$, $B_z$.

Flatness (Gaussian, 3):

$$\mathcal{F} (\tau) = \frac{\langle S^4_T \rangle}{\langle S^2_T \rangle^2}$$

Structure function:

$$S^p_T = \langle |V(t + \tau) - V(t)|^p \rangle$$

Slow wind more intermittent!

Intermittency at the bowshock

- Kolmogorov -5/3 spectra
- Non-Gaussians tails (F=3.5 – 4.6)

Four-point CLUSTER data

Narita et al., PRL., 97, 191101, 2006
Probability distribution functions

Helios: fast SW, \( V_x \) radial component of flow velocity

Non-Gaussian statistics at small scales!

Marsch and Tu, Annales Geophys., 12, 1127, 1994
Summary

- Solar wind is a weakly anisotropic turbulent magnetofluid.
- Alfvénic fluctuations dominate, with an admixture of weak compressive (magnetosonic) fluctuations.
- Turbulence develops towards Kolmogorov spectra, but intermittency prevails at small (below hourly) scales.
- Alfvén ratio, cross-helicity, anisotropy evolve radially, as does the average energy spectrum.
- Origin of the fluctuations: coronal sources for Alfvén waves, compressive waves from pressure imbalances and stream interactions, cascading by velocity shear.
- Structure functions and probability distribution reveal non-gaussian statistics.
Discontinuities and shocks

Continuity of the mass flux and magnetic flux:

\[ B_n = B_{1n} = B_{2n} \]
\[ G_n = \rho_1(V_{1n} - U) = \rho_2(V_{2n} - U) \]

U is the speed of surface in normal direction; \( B \) magnetic field vector; \( V \) flow velocity.
Mach number: \( M = V/C; \) C is the wave phase speed.

**Shock:** \( G \neq 0 \)

**Discontinuity:** \( G = 0 \)

Contact discontinuity (CD)
Index 1 upstream and 2 downstream;
\( B \) does not change across the surface of the CD, but \( \rho_1 \neq \rho_2 \) and \( T_1 \neq T_2 \).
Shocks (with mass flow)

Parallel shock

$\mathbf{B}$ is parallel to the normal $\mathbf{n}$ of the shock surface.

Perpendicular shock

$\mathbf{B}$ is perpendicular to the normal $\mathbf{n}$ of the shock surface.
Fast and slow shocks

Fast shock

B and V refract away from normal

Slow shock

B and V refract towards the normal
Possible geometries of shock normal and magnetic field

\[
\cos(\theta_{Bn}) = \frac{B \cdot n}{B}
\]
Discontinuities (no mass flow)

Tangential discontinuity (TD)

B is parallel to the surface of the TD, but its direction may change across it.

Rotational discontinuity (RD)

B is oblique to the surface of the RD and its direction changes across it.

Alfvén shock
The Helios spacecraft

Twin spacecraft for heliospheric physics in highly eccentric orbits with perihelion at 0.3 AU during the years 1974-1986.
Phase velocities of wave modes

Doppler shift:
\[ \omega' = \omega + k \cdot V_{sw} \]

Gurnett, 1978
Electrostatic waves

Plasma frequency

\[ \omega_{pe}^2 = \frac{4\pi e^2 n_e}{m_e} \]

Ion acoustic speed

\[ c_{ia}^2 = \frac{\gamma e k_B T_e}{m_i} \]

Debye length

\[ \lambda_D^2 = \frac{k_B T_e}{4\pi e^2 n_e} \]
Ion acoustic and Langmuir waves

Gurnett, 1991
Electric field power spectrum

- No power law but hump
- Abrupt decline at $f_p$ indicates electron Landau damping (absorption)
- Spectrum at frequencies between $10^3$ Hz and $5 \times 10^4$ Hz is mainly due to Doppler-shifted ion acoustic waves

Gurnett & Anderson, JGR 82, 632, 1977
Ion acoustic waves at a shock

\[ \omega = \omega_s + k \cdot V \]

\[ \omega_s = \frac{c_s k}{(1+k^2 \lambda_D^2)^{1/2}} \]
Electromagnetic waves

Ion gyrofrequency

\[ \omega_{gi} = \frac{q_i B}{m_i c} \]

Upper/ lower hybrid frequency

\[ \omega_{uh}^2 = \omega_{pe}^2 + \omega_{ge}^2 ; \quad \omega_{lh}^2 = \omega_{ge} \omega_{gi} \]
Whistler mode waves at a shock

\[ \omega_w = \omega_{pe} \left( \frac{k_c}{\omega_{ge}} \right)^2 \]

Gurnett et al., JGR 84, 541, 1979
Solar wind data in a magnetic cloud

Ion acoustic waves for $T_e/ T_p > 1$

Lin et al., in SW9, 673, 1999
Ulysses wave data - day 73 in 1995

Electron beam driven

Type III

Thermal noise

Langmuir waves

LF electrostatic waves

Whistler waves

Power in grey scale in dB above noise

McDowell and Kellog, 2001