# Intermittency in spiral Poiseuille flow

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Abstract. The results of an experimental study on intermittent spiral vortices observed in a counter-rotating Taylor-Couette system with an additional axial through flow, i.e. Spiral-Poiseuille flow, are presented. Convectively unstable upstream propagating spiral vortices appear in the laminar basic flow from an oscillatory instability and in general become absolutely unstable at higher inner cylinder Reynolds number. It is found that at Reynolds numbers above the absolute stability border the spiral vortices become unstable and a complex flow state showing intermittent bursts appears. The intermittent flow state is characterised by an irregular alternation between clearly distinguishable 'laminar' phases corresponding to up- and downstream propagating spiral vortices as well as propagating Taylor vortices. For a sufficiently high rate of axial through flow it is found that intermittency can occur directly from the convectively unstable regime of the upstream propagating spiral vortices.

# 1. Introduction

Taylor-Couette flow is a classical hydrodynamic system for the study of bifurcation events and the transition to turbulence [1, 2, 3]. It is the flow of a viscous liquid between two concentric coaxial rotating cylinders. In the classical setup the flow is confined in axial direction by solid end plates. This defines an aspect ratio  $\Gamma = \frac{L}{d}$ , i.e. the ratio of axial length L to the gap width d between the cylinders. It has become apparent in wide variety of studies that breaking the reflection symmetry with respect to the axial midplane of this closed hydrodynamic system is of fundamental importance for the organisation of complex dynamics in the flow [4, 5]. As one consequence of the additional axial through flow the character of the Taylor-Couette system changes from a closed to an open hydrodynamic system. In open systems generally the concept of convective and absolute stability plays a crucial role. For the Taylor-Couette system with stationary outer cylinder the convective and absolute stability boundaries have been determined numerically in [6, 7] and experimentally in [8, 9, 10]. Theoretical and numerical investigations on convective and absolute stability of spiral vortices which appear in Taylor-Couette flow from the primary instability in basic laminar flow in case of counter-rotating cylinders have been performed by [11, 12, 13, 14]. Experimentally the convective and absolute stability boundary of counter-rotating Taylor-Couette flow have been determined recently in [15]. As a further consequence of an additional axial through flow in the Taylor-Couette system the reflection symmetry of the flow is broken by an external perturbation. Studies on the organisation of complex nonlinear dynamics in open Taylor-Couette flow are much rare than in the closed counterpart and only performed in case of stationary outer cylinder so far. A linear stability analysis of the flow for several values of the rotation rate ratio between inner and outer cylinder has been investigated by Cotrell and Pearlstein [16] as well as by Meseguer and Marques [17].

However, Tsamaret and Steinberg [18], Lueptow et al. [9] and Bühler and Polifke [19] observed complex spatiotemporal dynamics in the nonlinear regime of this open flow in case of stationary outer cylinder. Furthermore several flow states such as non-wavy or wavy vortices as well as non-wavy or wavy helical vortices and random wavy vortices have been measured by Particle image velocimetry by Wereley and Lueptow [20], verified by numerical calculations of Hwang and Yang [21], for the case of stationary outer cylinder. In this study we present results from an experimental study on nonlinear dynamics of open Taylor-Couette flow in the case of counterrotating cylinders, i.e. of spiral Poiseuille flow. We focus on the stability of spiral vortices that appear for a sufficiently large rate of counter-rotation as the primary pattern from a Hopfbifurcation of the basic laminar flow.

# 2. Experimental Setup

The experimental setup consists of a viscous fluid confined in the gap between two independently rotating concentric cylinders. The inner cylinder is made of stainless steel having a radius of  $r_i = (12.50 \pm 0.01)$  mm. The outer cylinder consists of optically polished glass with a radius of  $r_o = (25.00 \pm 0.01)$  mm. As a working fluid silicone oil with the kinematic viscosity  $\nu = 11.2$  cSt is used. To ensure this value of kinematic viscosity to be constant within the accuracy for all measurements the temperature of the fluid is thermostatically controlled at  $(24.00 \pm 0.01)^o$ C. At top and bottom the fluid is confined by end plates which are held fixed in the laboratory frame. The end plates have a matrix of holes in a way that an axial through flow is enabled and radial and azimuthal velocity components of the through flow are avoided. They have identical shape to guarantee reflection symmetry of the system and the distance between them defines the axial length L of the flow which is adjustable within an accuracy of 0.01 mm up to a maximal axial length of 610 mm. Geometric parameters are the aspect ratio  $\Gamma = \frac{L}{d}$ , held fixed for all measurements at  $\Gamma = 22.8$ , with a gap width  $d = r_o - r_i$ , and a radius ratio  $\eta = \frac{r_i}{r_o}$  which is also held fixed to  $\eta = 0.5$  for all measurements. As control parameters serve the Reynolds number of the inner (i) and outer (o) cylinder,  $\operatorname{Re}_{i,o} = \frac{dr_{i,o}\Omega_{i,o}}{\nu}$ , where  $\Omega_{i,o}$  denote the rotation rates of the inner (i) and outer (o) cylinder, respectively. The Reynolds number of the axial



Figure 1. (a) Schematic plot of the experimental setup and (b) measured ( $\circ$ ) and analytically calculated (solid line) velocity profile of the axial through flow at  $Re_D = 1.5$  recorded at a distance of 12.5 mm from the inlet.

through flow is defined by  $\operatorname{Re}_D = \frac{d \langle v \rangle}{\nu}$ , where  $\langle v \rangle$  denotes the mean axial velocity. The axial through flow is enabled in both directions from the bottom up and from top down and its size is variable at  $0.1 \leq \operatorname{Re}_D \leq 40$ . A schematic plot of the Taylor-Couette system with axial through flow is shown in figure 1 (a). We utilised Laser Doppler velocimetry (LDV) for the measurements of the flow velocity. The measured velocity profile ( $\circ$ ) is in good agreement with the analytically calculated Poiseuille profile, in particular even at a distance of 12.5 mm from the inlet, as illustrated in figure 1 (b). All following measurements of the axial flow velocity are recorded at the radial distance of 1.3 mm from the inner cylinder and an axial height of 160 mm. In addition flow visualisation has been done in order to determine the spatial pattern of the flow. For typical flow pattern of Spiral-Poiseuille flow see figure 2.

# 3. Results

As shown recently by Langenberg et al. [15] a reasonable agreement with respect to the convective stability boundary and a good agreement with the absolute stability boundary of spiral vortices in counter-rotating Taylor-Couette flow can be found between numerics [12] and experiments. The spiral vortices, that appear as the primary pattern from a Hopf-bifurcation at sufficiently low rate of through flow, always propagate upstream, i.e. in the direction of the external through flow. However, downstream propagating spiral vortices as well as propagating Taylor vortices are also observed in counter-rotating Taylor-Couette flow with axial through flow. In order to characterise the different flow states photographs from a laser light-sheet measurement in the (r,z)-plane of upstream- as well as downstream propagating sprirals are depicted in figure 2 (a) and (b). Here, the axial through flow is directed from the left to the right. All photographs are recorded in the axial middle of the system with  $\Gamma = 22.8$ . The photos in figure 2 (a) and (b) are obtained at  $\text{Re}_i = 130$  and  $\text{Re}_o = -100$  but at different through flow Reynolds numbers  $\text{Re}_D = -1.5$  for  $\text{SPI}\uparrow$  (a) and  $\text{Re}_D = +1.5$  for  $\text{SPI}\downarrow$  (b). The third photograph, depicted in figure 2 (c) is recorded at  $\text{Re}_i = 110$ ,  $\text{Re}_o = -50$  and  $\text{Re}_D = +3.0$ . The particles added to the flow have a diameter of 8  $\mu$ m and have been used for both LDV and flow visualisation with a laser light-sheet.



**Figure 2.** Photographs from a laser light-sheet measurement in the (r,z)-plane of the flow in the axial midplane: (a) upstream propagating spirals (SPI<sup> $\uparrow$ </sup>) for Re<sub>D</sub> = -1.5, b) downstream propagating spirals (SPI<sup> $\downarrow$ </sup>) for Re<sub>D</sub> = +1.5 both recorded at Re<sub>i</sub> = 130 and Re<sub>o</sub> = -100 and (c) propagating Taylor vortices (PTV) measured at Re<sub>i</sub> = 110, Re<sub>o</sub> = -50 and Re<sub>D</sub> = +3.0.

In the nonlinear regime of the flow above the absolute stability boundary of upstream propagating spirals (SPI $\uparrow$ ) a complex phenomena of stability exchange could be observed for certain Reynolds numbers of through flow Re<sub>D</sub>. An irregular spatial as well as temporal

alternation of different flow states, like propagating Taylor vortices (PTV), up- (SPI $\uparrow$ ) and downstream propagating spirals (SPI $\downarrow$ ), is one of the representative characteristics of the flow in this regime. A time series of the axial velocity, which is recorded after a transient of 12



**Figure 3.** (a) Typical bursts taken from 'intermittent' time series. An alternation of 'laminar' phases of propagating Taylor vortices (PTV) as well as upstream (SPI<sup>↑</sup>) and downstream propagating spirals (SPI<sup>↓</sup>) occurs. The time series is recorded at  $\text{Re}_i = 125$ ,  $\text{Re}_o = -100$  and  $\text{Re}_D = 3.75$ . (b) Histogram of relative frequency vs. length of oscillation period obtained from total intermittent time series. The total recording time is approximately 24 hours after a transient of 12 hours.

hours, is depicted in figure 3 (a). This time series has a length of 670 s, whereas all control parameters are held fixed, i.e.  $\operatorname{Re}_i = 125$ ,  $\operatorname{Re}_o = -100$  and  $\operatorname{Re}_D = 3.75$  for the whole recording time. The intermittent character of the flow is obvious from figure 3 (a). Three different types of oscillations can be distinguished already by eye from the time series in figure 3 (a) alternating irregularly between these three oscillatory regimes. Detailed investigations of the flow regime by simultaneous flow visualisation and LDV measurements allow a unique distinction between the different flow regimes. In this particular example SPI occurs for about 80 s, then replaced by PTV for approximately 300 s, sequencing SPI<sup>↑</sup> for about 40 s and PTV again for 170 s, and finally interrupted by SPI $\uparrow$ . The oscillation periods can be measured and three different flow regimes can be classified on a quantitative basis by a histogram of the frequency of oscillation period. One example is illustrated in figure 3 (b). The oscillation periods have been computed considering the angle of the Hilbert-transformation of the time series. Classical methods like Fourier-transformation are less practical because of the partially short-time phases of the different flow regimes. The oscillation periods of SPI↑ is smallest with about 2.4 s and SPI $\uparrow$  do occur with lowest frequency (1 %) in the flow. SPI $\downarrow$  is clearly distinguishable from SPI $\uparrow$ and results in a very narrow but high peak centred around 3.9 s. This results from a frequent occurrence of this flow regime in this flow state. In contrast the peak of period of PTV is much wider than the SPI<sub>+</sub>-peak but lower and is located around 10.5 s. Nevertheless, as already indicated by the time series in figure 3 (a) PTV also occur quite frequently in the flow but with a much less determined oscillation period. It can be concluded that the intermittent flow state is characterised by an irregular alternation between 'laminar' phases of  $SPI\uparrow$ ,  $SPI\downarrow$ , and PTV.

In particular since other periods, not related to these three flow regimes, could not be observed.



Figure 4. Stability diagram for numerically (dotted line, from [12]) and experimentally (•) determined convective instability of basic laminar Couette-Poiseuille flow (CPF); numerically (solid line, from [12]) and experimentally ( $\circ$ ) determined absolute stability boundary of SPI $\uparrow$ ; experimentally determined transition to intermittency ( $\diamond$ ). Points (a), (b) and (c) refer to time series shown in figure 5.

Intermittent flow states like the one characterised by time series shown in figure 3 (a) can be observed in a wide parameter regime of this hydrodynamic system. The stability diagram of SPI $\uparrow$  at  $\Gamma = 22.8$  and  $\text{Re}_o = -100$  in the ( $\text{Re}_i, \text{Re}_D$ )-plane is shown in figure 4. In the figure the dotted line represents the numerically calculated convective stability boundary for cylinders with an infinite axial length [12]. Here, the basic Couette-Poiseuille flow (CPF) becomes convectively unstable and SPI<sup>↑</sup> appears under the assumption of infinite axial length. The convective stability boundary in the experiment  $(\bullet)$  is taken to be the lowest value of inner cylinder Reynolds number  $\operatorname{Re}_i$ , where the frequency peak of the upstream propagating spirals is detectable in the power spectrum. The deviation between experimentally determined and numerically calculated stability lines is related to the systematic difference between experiment and numerics resulting from the finite length of the experimental system (see [15]). The absolute stability boundary which appear for higher  $\operatorname{Re}_i$  has been calculated numerically for the case of infinite axial length of the cylinders (solid line) by [12]. It has been confirmed experimentally between  $3.5 \leq Re_D \leq 5.3$ (o). For Reynolds number  $5.3 < Re_D \leq 6.0$  the absolute stability boundary is superseded by a direct transition from convective SPI $\uparrow$  to intermittency (dashed line,  $\diamond$ ). A transition to intermittency is observed for the entire parameter regime  $3.5 \le Re_D \le 6$  shown in figure 4 but



in case of  $\text{Re}_D \leq 5.3$  this transition is above the absolute stability boundary of SPI $\uparrow$ . Three

**Figure 5.** Typical time series with corresponding power spectra recorded at  $\text{Re}_o = -100$  and  $\text{Re}_D = 6.00$ : (a) convective unstable SPI $\uparrow$  ( $\text{Re}_i = 108.57$ ), intermittency (b) below ( $\text{Re}_i = 114.93$ ) and (c) above ( $\text{Re}_i = 118.91$ ) the numerically determined absolute stability boundary of SPI $\uparrow$ .

different time series, recorded all at  $\text{Re}_o = -100$  and  $\text{Re}_D = 6.0$ , and corresponding power spectra are depicted in figure 5. They have been measured at different Reynolds numbers of the inner cylinder  $\operatorname{Re}_i$  representing the three different flow regimes. Their locations in the stability diagram are indicated by (a), (b) and (c) in figure 4. The time series in figure 5 (a) has been measured at  $\text{Re}_i = 108.57$  from a convective unstable flow regime (see [15]). In particular a peak of SPI<sup>↑</sup> can be observed at 0.55 Hz in the corresponding power spectrum, broadened in comparison to a typical peak measured in the absolutely unstable regime. In figure 5 (b) bursts can be observed in time series recorded at  $\operatorname{Re}_i = 114.93$  which is below the numerically calculated absolute stability boundary. From the corresponding power spectrum a broadened peak at 0.22 Hz additional to the 0.55 Hz peak from SPI↑ can be observed. Due to the large through flow ( $\text{Re}_D = 6.0$ ) the frequency of SPI $\downarrow$  and of PTV have shifted to 0.05 Hz and to 0.22 Hz, respectively. Therefore the bursting can be related to short periods of PTV-bursts alternating with relatively long periods of  $SPI\uparrow$ . In addition to this sidebands can be observed in the corresponding power spectrum. These sidebands are the result of the almost periodic but still irregular alternation of PTV- and SPI<sup>-</sup>phases in this regime. This alternation has a frequency of 0.014 Hz, which corresponds to a period of approx. 71 s. Note that the phases of SPI $\uparrow$  are almost periodic, but irregular for PTV. Above the numerically calculated absolute stability boundary the flow is still characterised by an intermittent alternation between SPI $\uparrow$  and PTV, which is not periodic at all. The irregular behaviour of intermittency in this regime can be seen from the time series depicted in figure 5 (c) as well as from the corresponding power spectrum with its broadband character and the broadened peaks of PTV and SPI $\uparrow$ .

#### 4. Conclusions

In counter-rotating Taylor-Couette flow with axial through flow, i.e. in spiral Poiseuille flow, the experimentally determined transition from the basic laminar flow to upstream propagating spirals (SPI $\uparrow$ ) is in reasonable agreement for convective and in good agreement for the absolute stability boundary with the numerical calculations (see also [12, 15]) as long as the Re<sub>D</sub> is sufficiently low (Re<sub>D</sub>  $\leq$  5.3). In this study we have shown experimentally that above this particular rate of through flow the absolute stability boundary of upstream propagating SPI $\uparrow$ can be superseded by a transition to intermittency. The intermittent flow state is observed in a wide parameter range of Re<sub>i</sub> and Re<sub>D</sub> but for Re<sub>D</sub>  $\leq$  5.3 the transition occurs only well above the absolute stability boundary of SPI $\uparrow$ . The intermittent flow is characterised by an irregular alternation between up- and downstream propagating spirals as well as propagating Taylor vortices. The observed intermittent flow state and the exchange between the absolute stability and the intermittency illustrates the complex nonlinear dynamics of spiral Poiseuille flow and will be subject of further research.

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#### 5. References

- [1] T. Mullin. The nature of chaos. Oxford University Press, Oxford (England), 1993.
- [2] R. Tagg. Nonlin. Sc. Today, 4(3):1–25, 1994.
- [3] J. Abshagen, G. Pfister, and T. Mullin. Phys. Rev. Lett., 87(22):1-4, 2001.
- [4] T.B. Benjamin. Proc. R. Soc. Lond. A, 359:27, 1978.
- [5] J. Abshagen, O. Meincke, G. Pfister, K.A. Cliffe, and T. Mullin. J. Fluid Mech., 476:335–343, 2003.
- [6] H.W. Müller, M. Lücke, and M. Kamps. Europh. Lett., 10:451-456, 1989.
- [7] A. Recktenwald, M. Lücke, and H.W. Müller. Phys. Rev. E, 48(6):4444-4454, 1993.
- [8] K.L. Babcock, G. Ahlers, and D.S. Cannell. Phys. Rev. E, 50(5):3670-3692, 1994.
- [9] R.M. Lueptow, A. Docter, and K. Min. Phys. Fluids A, 4(11):2446–2455, 1992.
- [10] A. Tsamaret and V. Steinberg. Phys. Rev. E, 49(2):1291-1308, 1994.
- [11] D.I. Takeuschi and D.F. Jankowsi. J. Fluid Mech., 102:101–126, 1980.
- [12] A. Pinter, M. Lücke, and C. Hoffmann. Phys. Rev. E, 67(2):1-15, 2003.
- [13] P. Büchel, M. Lücke, D. Roth, and R. Schmitz. Phys. Rev. E, 53(5):4764–4777, 1996.
- [14] J.-M. Chomaz. Phys. Rev. Lett., 69(13):1931–1934, 1992.
- [15] J. Langenberg, M. Heise, J. Abshagen, and G. Pfister. Theo. Comp. Fluid Dyn., 18(97), 2004.
- [16] D.L. Cotrell and A.J. Pearlstein. J. Fluid Mech., 509:331–351, 2004.
- [17] A. Meseguer and F. Marques. J. Fluid Mech., 455:129–148, 2002.
- [18] A. Tsamaret and V. Steinberg. Phys. Rev. E, 49(5):4077–4086, 1994.
- [19] K. Bühler and N. Polifke. NATO ASI Series, Series B, 225(3):21–29, 1989.
- [20] S.T. Wereley and R.M. Lueptow. Phys. Fluids, 11:3637–3649, 1999.
- [21] J.-Y. Hwang and K.-S. Yang. Computers and Fluids, 33(1):97–118, 2004.