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Problems for *Physik der Materie III*

Due by May 22, 2019

Series 5: Specific heat and Drude theory of metals

5.1 Specific heat capacity of a monoatomic chain

Consider a one-dimensional linear chain of N identical atoms.

(1) Use the Debye model to calculate the specific heat capacity c_v at temperatures much smaller than the Debye temperature $\Theta_D = \hbar\pi v/k_B a$, where v is the velocity of sound, a is the atomic spacing and k_B is the Boltzmann constant. Compare the result with the corresponding specific heat of a three-dimensional crystal. Use $\int_0^\infty x^2 e^x/(e^x-1)^2 dx = \pi^2/3$.

(2) Typical values for the velocity of sound and the atomic spacing are $v = 5 \cdot 10^3$ m/s and $a = 2.5$ Å, respectively. Calculate the Debye temperature and the Debye frequency ω_D. Also calculate c_v/Nk_B at liquid Helium temperature (4.2 K), where c_v is the specific heat determined in (1).

5.2 Drude model

Neglecting electron-electron interaction, the Drude model assumes that conduction electrons move uniformly in straight lines between two collisions with the ions of the metal. The probability of an electron to undergo a collision per unit time is $1/\tau$, where τ is the "relaxation time". Collisions are assumed to be instantaneous events that abruptly alter the velocity $\mathbf{v}$ of an electron, and are accounted for by a friction term $-m\mathbf{v}(t)/\tau$ in the equation of motion, where m is the electron mass.

(a) External fields acting on the electrons are taken into account via Newton's laws of motion. Write down the Drude equation of motion for an electron when both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ are applied.

(b) Show that in the steady state regime where the current is independent of time ($d\mathbf{j}/dt = \mathbf{0}$), the current density $\mathbf{j}$ is

$$\mathbf{j} = \frac{\tau n e^2}{m}\mathbf{E} - \frac{\tau e}{m}\mathbf{j} \times \mathbf{B},$$

where n is the electron density.

(c) Taking $\mathbf{B}$ to be along the z-axis and using the cyclotron frequency $\omega_c = eB/m$, determine the conductivity tensor $\bar{\boldsymbol{\sigma}}$ as defined by the relation $\mathbf{j} = \bar{\boldsymbol{\sigma}} \cdot \mathbf{E}$. Simplify the tensor components as to only contain ω_c, τ, and $\sigma_o = \tau n e^2/m$. Also show that for most metals which have relaxation times τ of the order of 10^{-14} s, the anisotropy of $\bar{\boldsymbol{\sigma}}$ is small even for magnetic fields of $B = 1$ T.