Prof. Dr. R. Berndt
T. Jasper-Tönnies

Problems for *Physik der Materie III*

Due by June 26, 2019

Series 9: Fermi surface of a metal, Ewald construction, and intrinsic semiconductors

9.1 Fermi surface of gold

The periodicity observed in the de Haas-van Alphen effect is linked to an extremal cross-sectional area A_k of the Fermi surface, which is perpendicular to $\boldsymbol{B}$.

(a) Assuming that Au can be approximated by a three-dimensional free electron Fermi gas, estimate the extremal area A_k of the Fermi surface using an electron density of $n_{el}^{Au} = 5.90 \times 10^{22}\,\mathrm{cm}^{-3}$.

(b) From the equality of centrifugal and Lorentz forces for free electrons in a magnetic field, derive a relation between the "radii" of the extremal orbits in real and reciprocal space. Using the result of (a), calculate the extremal area A_r in real space for Au in a field B of 1 T.

(c) Experiments on Au show an oscillation period of $\Delta(1/B) = 1.95 \times 10^{-5}\,\mathrm{T}^{-1}$ along the [001]-direction and a superposition of two periods of $2.05 \times 10^{-5}\,\mathrm{T}^{-1}$ and $6.0 \times 10^{-4}\,\mathrm{T}^{-1}$ along the [111]-direction.
Calculate the corresponding extremal areas A_k in k-space and discuss the results in terms of the "real" Fermi surface shown in Fig. 1.

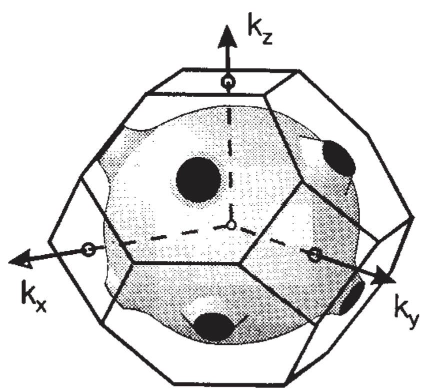

Figure 1: Fermi surface of gold

9.2 Ewald construction

X-rays with an energy of 7 keV are used to analyze a simple orthorhombic lattice ($a = 0.6$, $b = 0.8$, $c = 0.4$ nm). The x-rays propagate in the (001) plane and also the detector scans in the (001) plane. The crystal is being rotated around the $\langle 001 \rangle$ direction. Sketch the Ewald construction for this scenario. Determine the number and the scattering angles of the scattered beams using a computer and a programming language of your choice.

9.3 Conductivity of an intrinsic semiconductor

Consider an intrinsic Ge crystal at liquid nitrogen temperature ($T = 77$ K). Assume an average electron carrier concentration $n = 10^{12}\,\mathrm{cm}^{-3}$ due to optical excitation across the band gap. At this temperature, the mobilities of electrons and holes are assumed to be the same with $\mu_n = \mu_p = \mu \approx 5 \times 10^3\,\mathrm{cm}^2/\mathrm{Vs}$.
Determine the electrical conductivity σ and, for a cube of an edge length of 1 cm, the current I between two opposite faces at a voltage of 100 V.