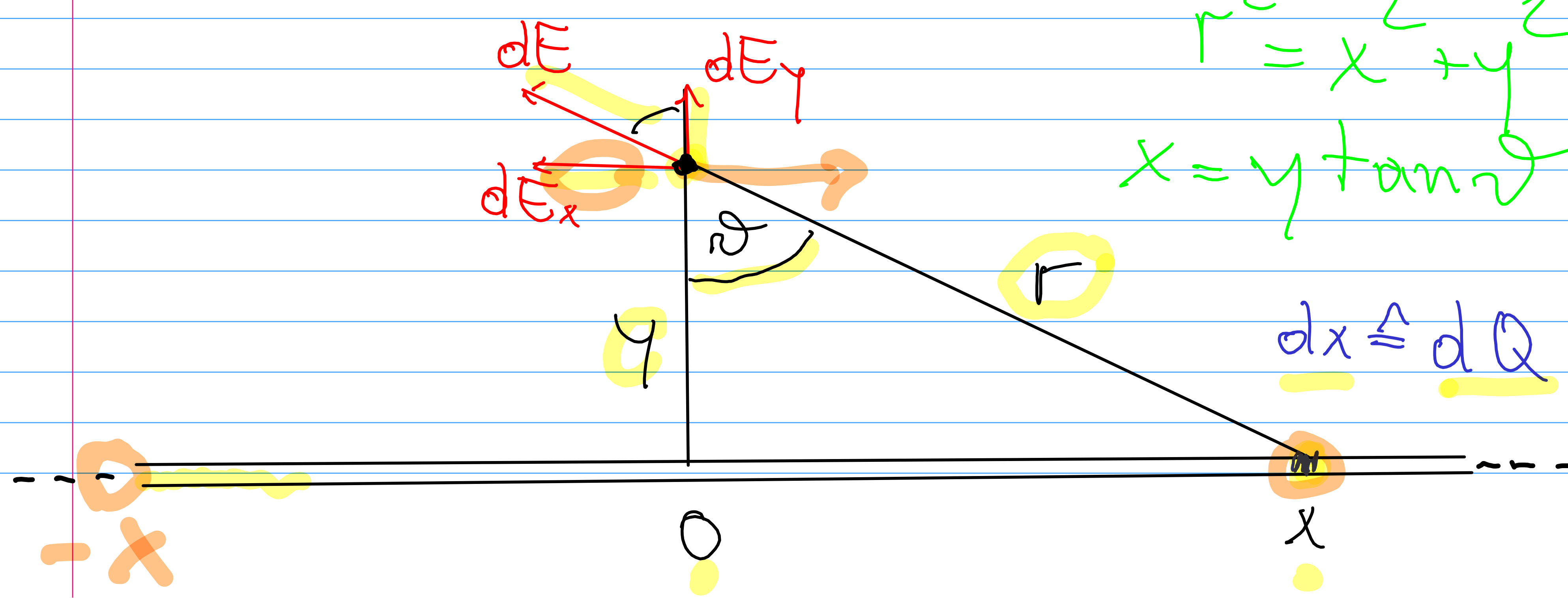
Kontinuierliche Verteilungen

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2} \quad ; \text{ integrieren!}$$

Beipiel 1: langer Draht



$$r^2 = x^2 + y^2$$

$$x = y \tan\vartheta$$

$$dx \mathrel{\hat{=}} dQ$$

$$dQ = \lambda \cdot dx \qquad [\lambda] = C/m$$

$$|dE| = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{x^2 + y^2}$$

$$dE_x = -dE \sin\vartheta \quad \Big| \quad E_x = 0$$

$$dE_y = dE \cos\vartheta$$

$$E_y = \int dE_y = \int_{-\infty}^{\infty} \cos\vartheta \, dE = 2\int_0^{\infty} \cos\vartheta \, dE$$

$$E_y = \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\infty}\cos\vartheta\,\frac{dx}{x^2+y^2}$$

xournal

$$x = y\tan\vartheta \Rightarrow \frac{dx}{d\vartheta} = y\,\frac{1}{\cos^2\vartheta}$$

$$\frac{1}{x^2+y^2} = \frac{1}{y^2\tan^2\vartheta + y^2} = \frac{1}{y^2}\,\frac{1}{1+\tan^2\vartheta}$$

$$= \frac{1}{y^2}\,\frac{\cos^2\vartheta}{\sin^2\vartheta+\cos^2\vartheta} = \frac{1}{y^2}\cos^2\vartheta$$

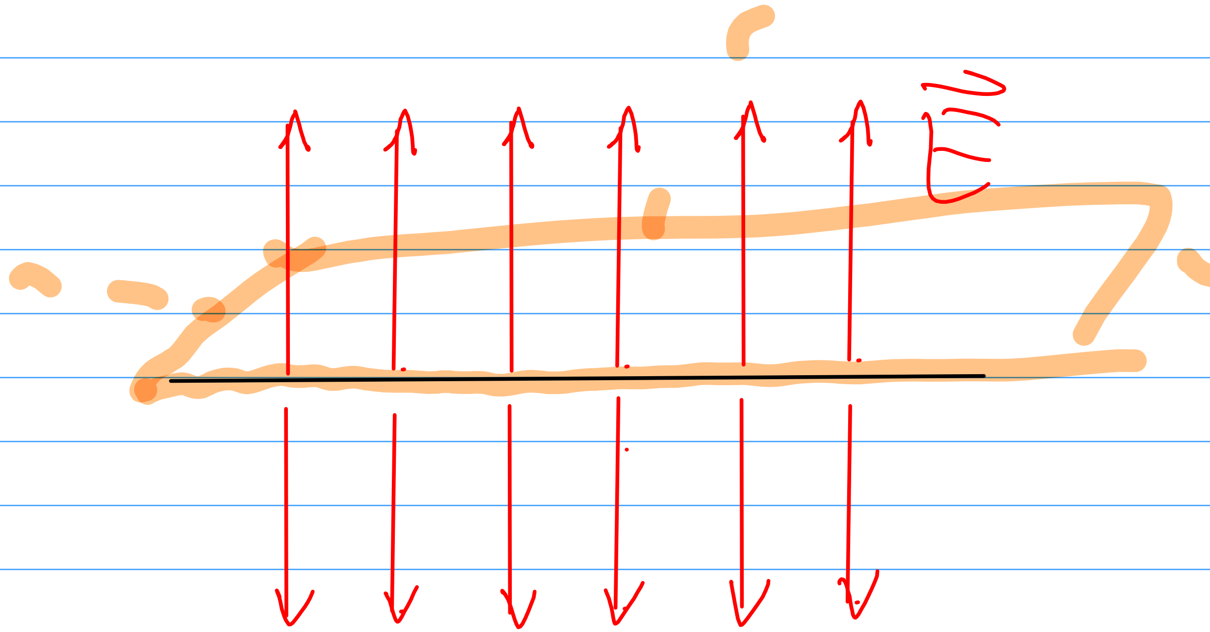
$$E_y = \frac{\lambda}{2\pi\varepsilon_0 y} \int_{\vartheta=0}^{\pi/2} \cos\vartheta \, d\vartheta = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{y}$$

Beispiel 2 Platte mit Ladungsdichte σ

Übungsaufgabe

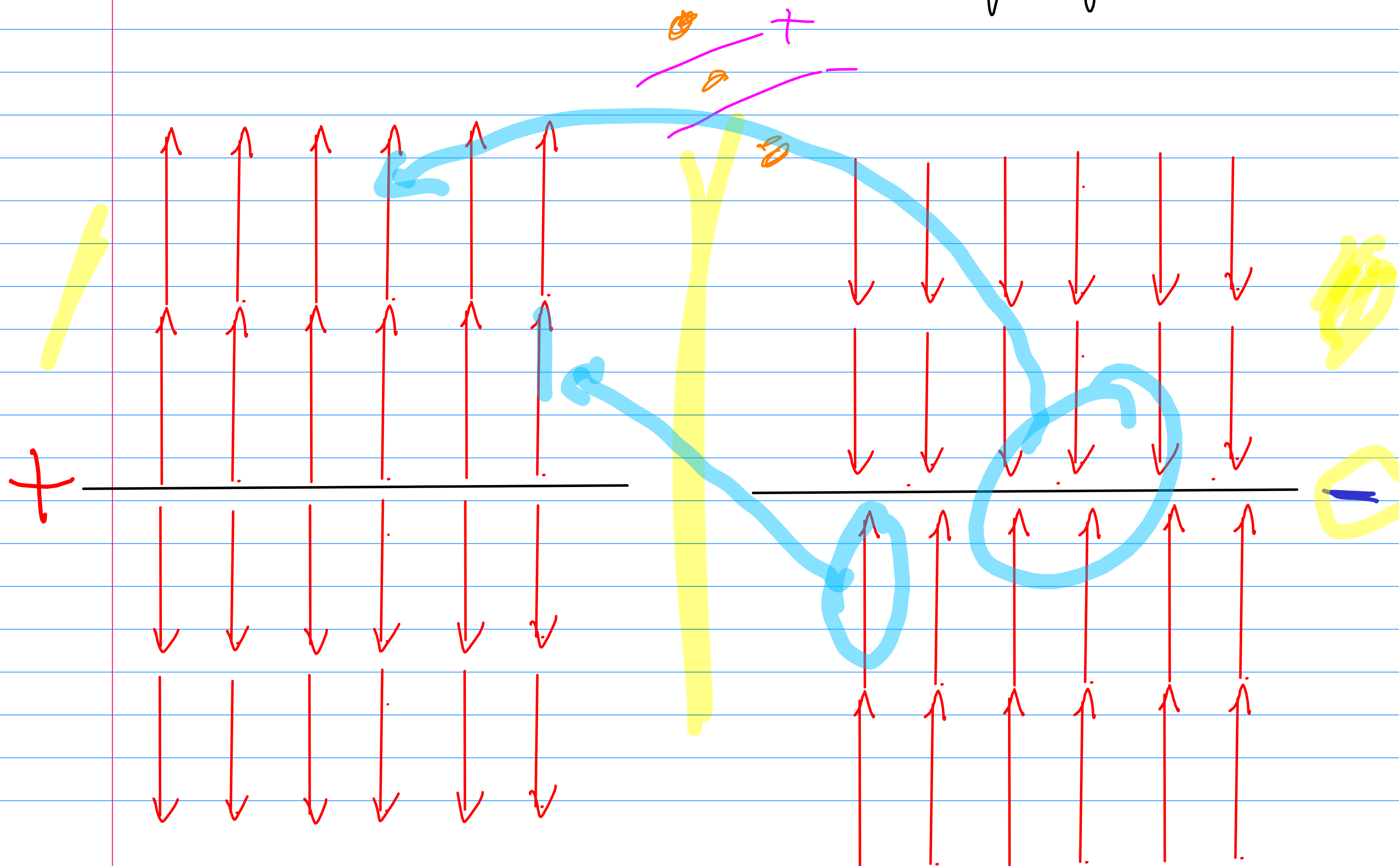
Ergebnis:

$$E = \frac{\sigma}{2\varepsilon_0}$$

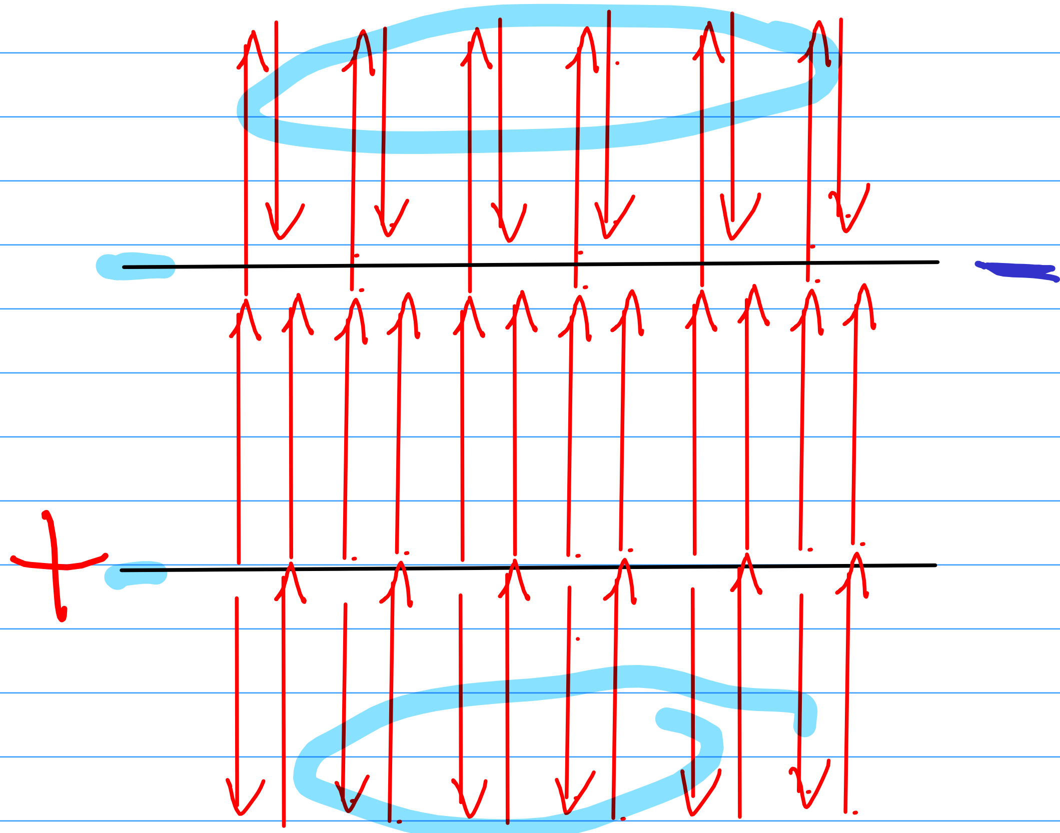


$$[\sigma] = C/m^2$$

Plattenkondensator: Superposition!



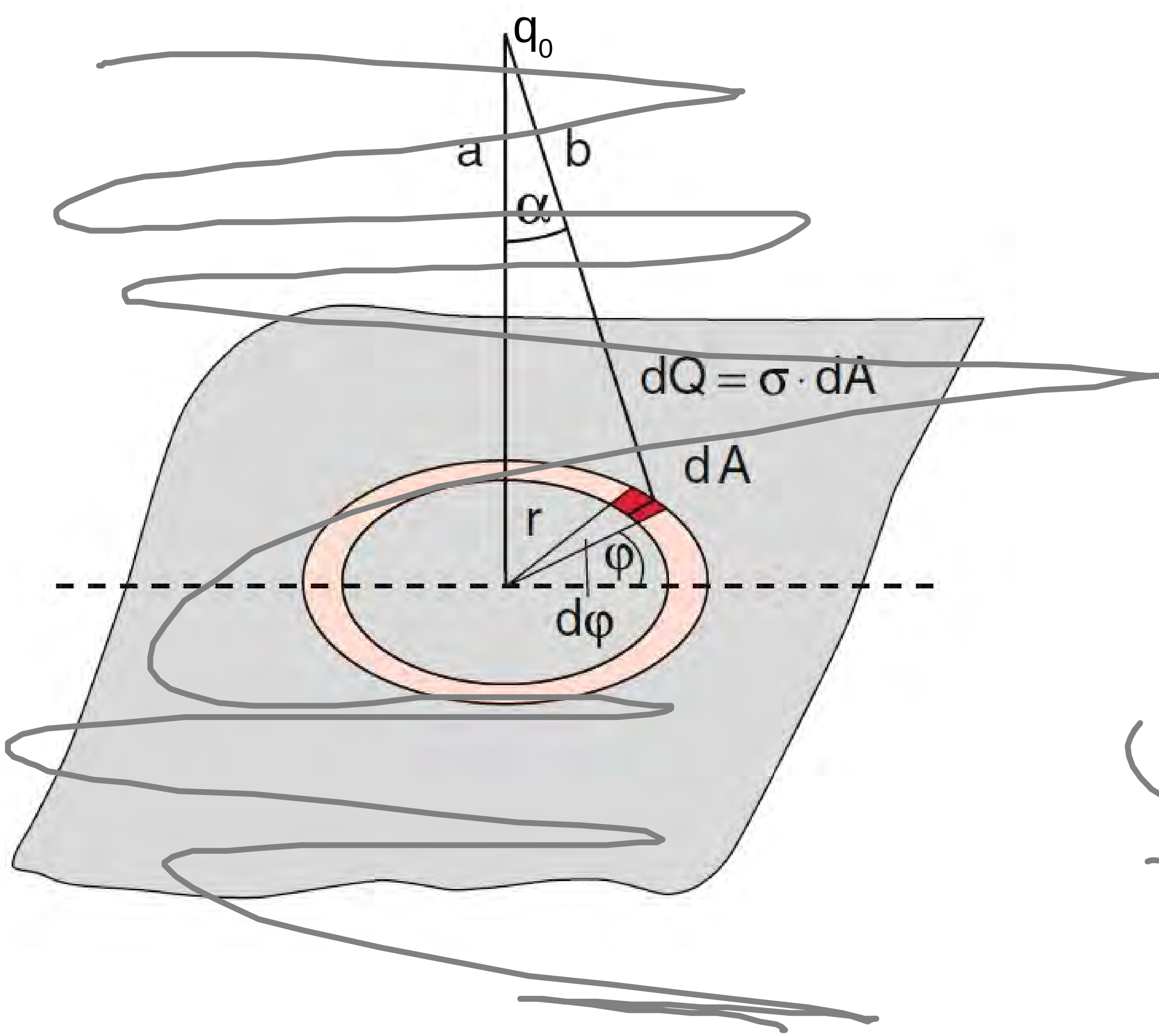
$$E = E_+ + E_- = \ldots$$



$$E = 0$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = 0$$



skip!

Braunsche Röhre

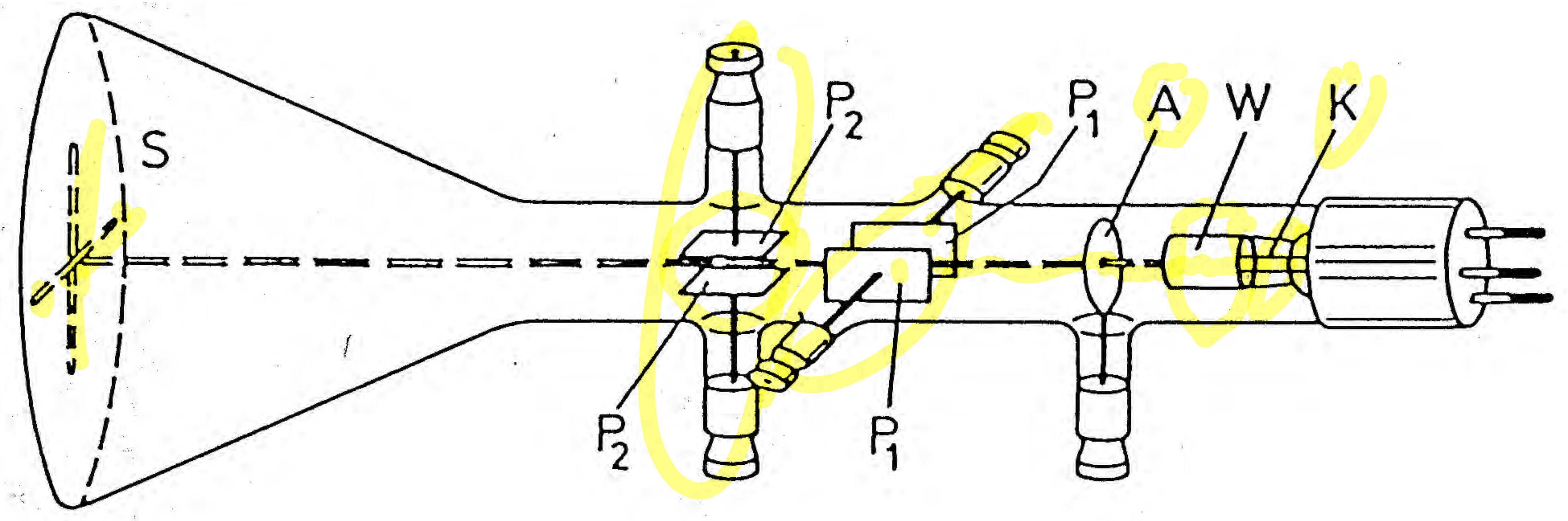
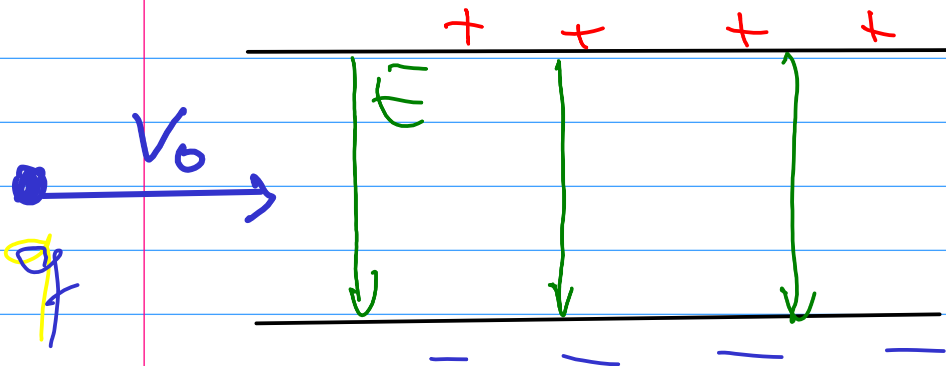
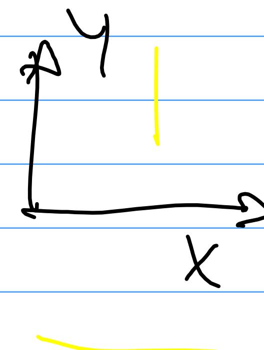


Abb. 502 Alte gaskonzentrierte Oszillographenröhre (Braunsche Röhre) mit Glühkathode K, Wehneltzylinder W, Anode A, Ablenkplatten P und Leuchtschirm S; (aus der Zeit um 1930)

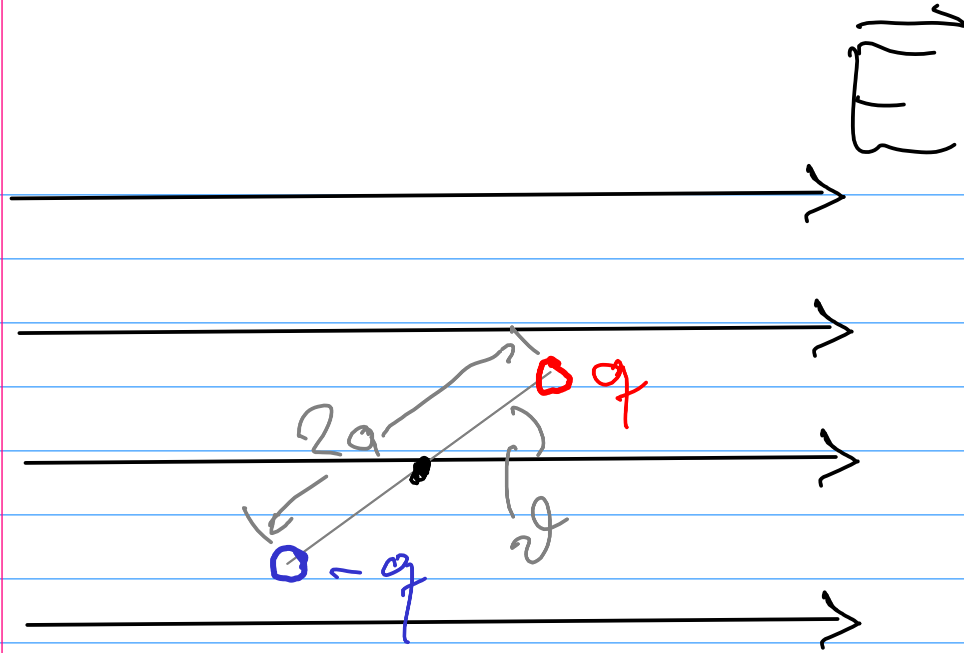
horizontaler Wurf!





$$x = v_0 t$$

$$y = qE \frac{x^2}{2 m v_0^2} = \frac{1}{2} \frac{F}{m} \frac{x^2}{v_0^2} = \frac{1}{2} a t^2$$



$$\vec{F} = \vec{F}_- + \vec{F}_+$$
$$= 0$$

Keine Translation

Drehmoment: $\tau = 2Ea\sin\vartheta$

$$= 2E\varphi a\sin\vartheta$$

$$\tau = pE\sin\vartheta$$

$$\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$$

$$W = \int dW = \int_{\Theta_0}^{\Theta} \tau \, d\vartheta = \int_{\Theta_0}^{\Theta} pE \sin\vartheta \, d\vartheta$$

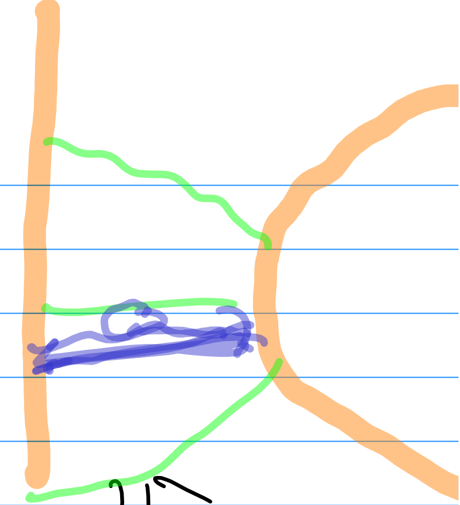
$$= -p\cos\vartheta \Big|_{\Theta_0}^{\Theta} E \quad ; \text{ sei } \Theta_0 = 90^\circ$$

$$\underline{W} = -pE\cos\Theta = \underline{-\vec{p}\cdot\vec{E}}$$

$$F = q(E_0 + \Delta E) - q(E_0 - \Delta E)$$

$$= q\, 2\Delta E$$

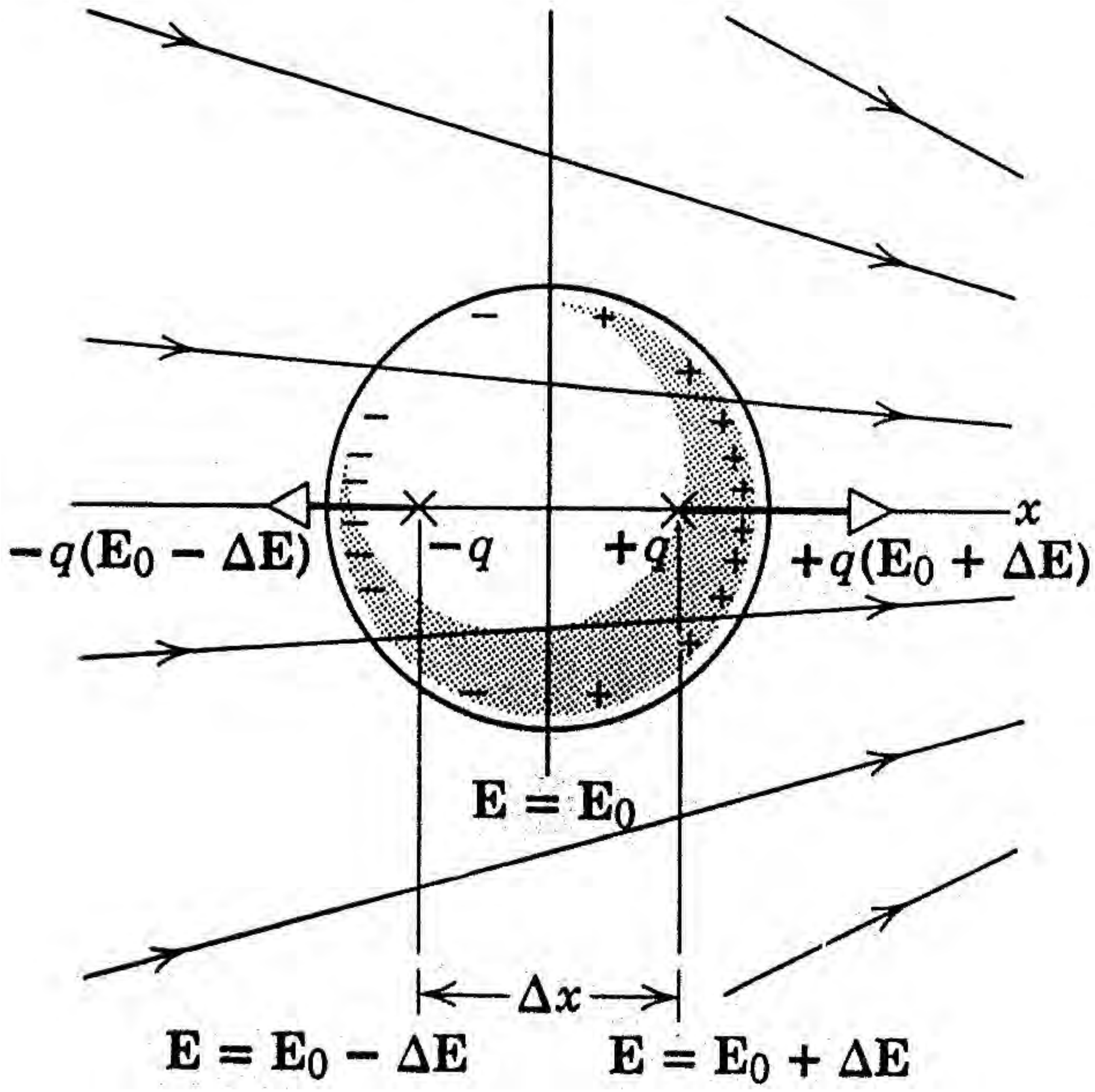
$$= q \frac{\Delta x}{\Delta x} 2\Delta E = p \frac{\Delta E}{\Delta x} \approx p \frac{dE}{dx}$$



$$\vec{F} = \frac{d}{dx}(\vec{p}\cdot\vec{E})$$

allgm: $\boxed{\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E})}$

Inhomogenes Feld: Kraft auf ~~Influenz~~dipol



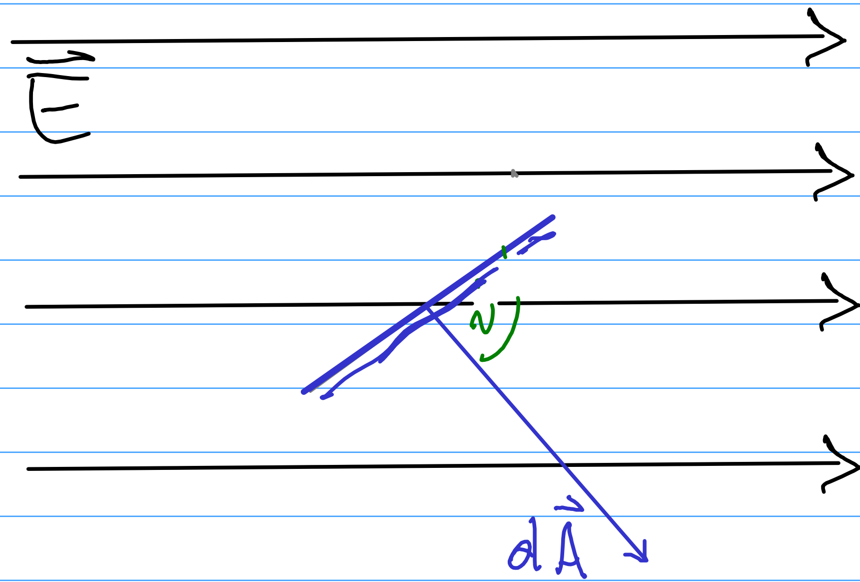
A hydrogen atom is composed of a nucleus containing a single proton, about which a single electron orbits. The electric force between the two particles is 2.3×10^{39} greater than the gravitational force! If we can adjust the distance between the two particles, can we find a separation at which the electric and gravitational forces are equal?

A 1. Yes, we must move the particles farther apart.

B 2. Yes, we must move the particles closer together.

C 3. no, at any distance

Gauß'scher Satz

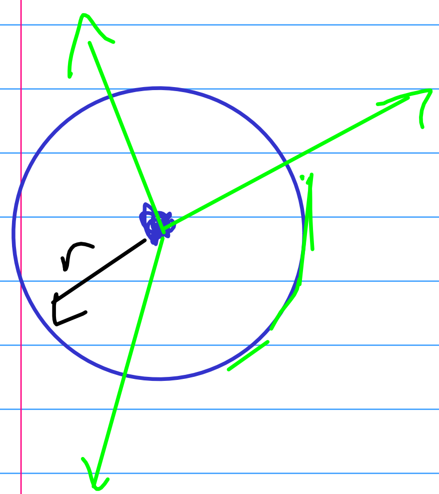


elektrischer Fluss

$$d\Phi_E = E\,dA\cos\vartheta$$
$$= \vec{E}\,d\vec{A}$$

$$\Phi_E = \int_{\text{Fläche}} \vec{E}\,d\vec{A}$$

Punktladung: $E = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} q$



$$\Phi_E = E \cdot 4\pi r^2 = \boxed{\frac{q}{\varepsilon_0} = \Phi_E}$$

Gaußscher Satz

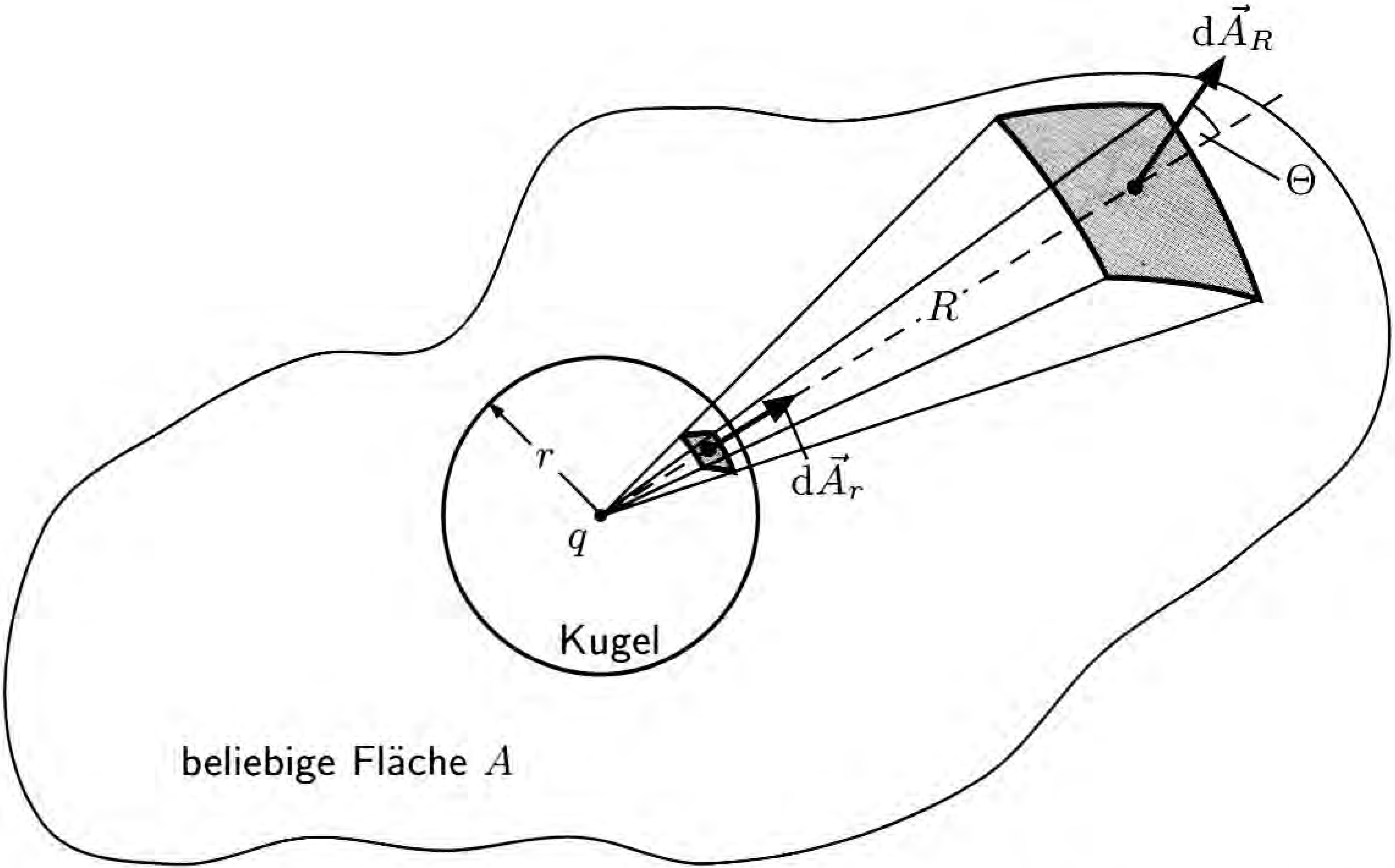
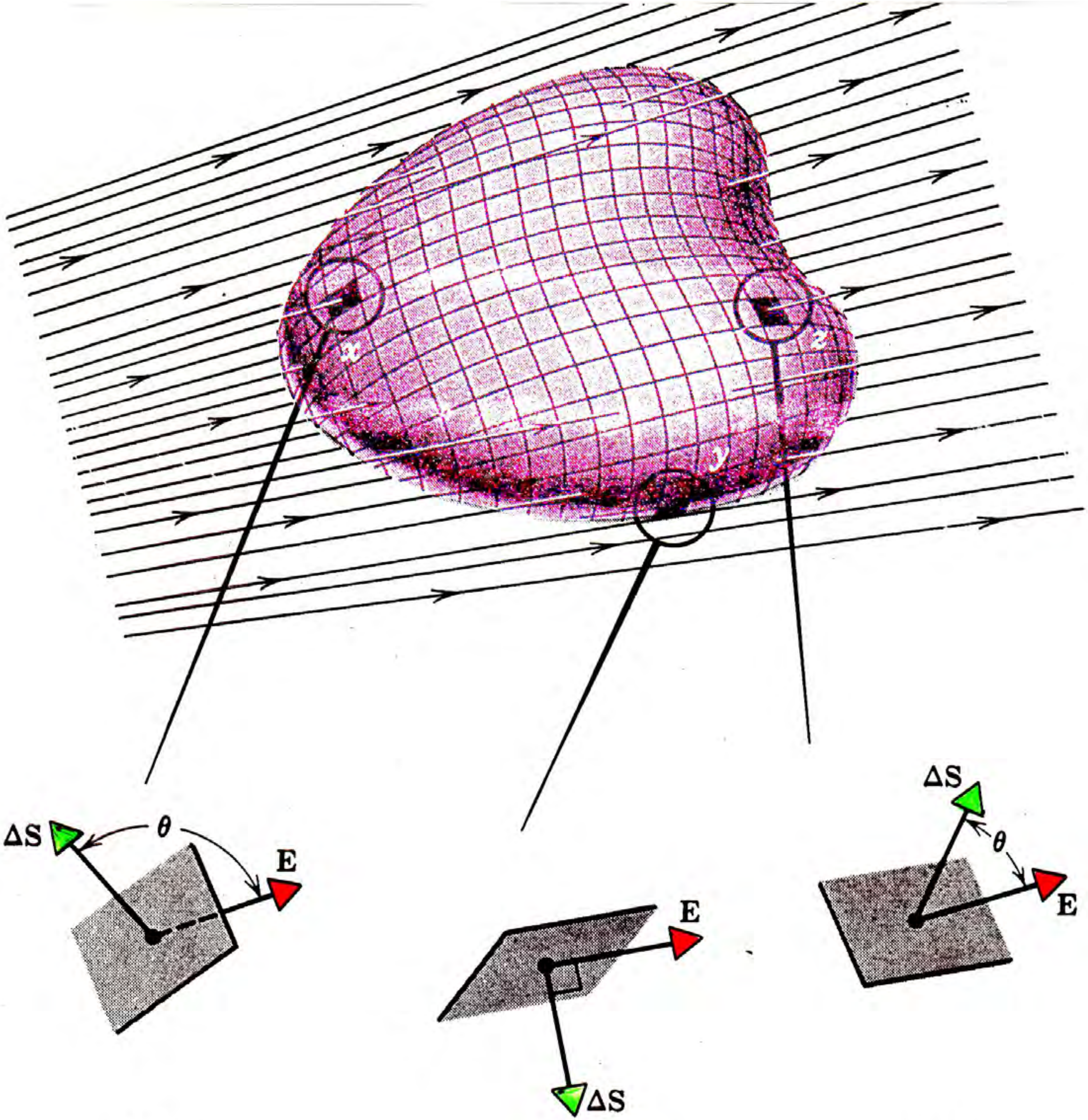


Bild 2.22: Projektion des Flächenelementes $\mathrm{d}\vec{A}_r$ der Kugeloberfläche auf eine beliebige Fläche A.



ΔS
θ
E
(x)
E
ΔS
(y)
ΔS
θ
E
(z)

$d\tilde{A}_r$ projeziere auf $d\tilde{A}_R$, Abstand R

Neigung Θ

$$dA_R = \frac{R^2}{r^2}\, dA_r \frac{1}{\cos\Theta}$$

Fluss durch $d\tilde{A}_R$:

$$\underline{d\Phi_E} = \tilde{E}_R \cdot d\tilde{A}_R = \left(E_r \frac{\cancel{r^2}}{\cancel{R^2}}\right)\left(\frac{\cancel{R^2}}{\cancel{r^2}} dA_r \frac{1}{\cancel{\cos}\Theta}\right)\cancel{\cos}\Theta$$

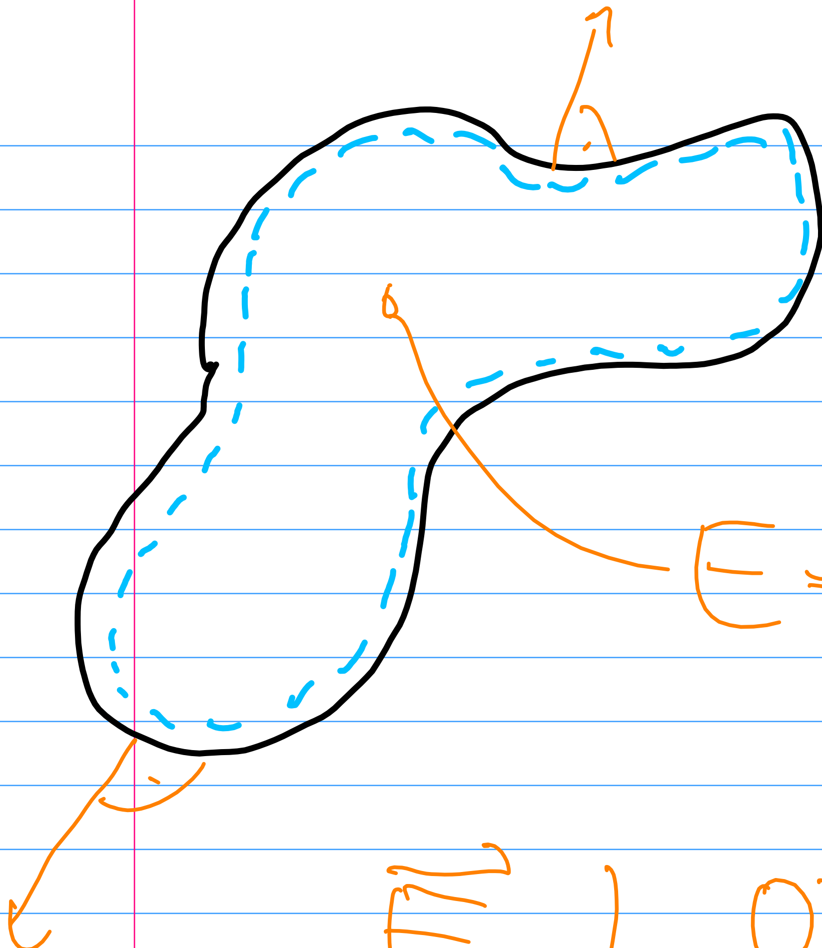
$$= \underline{\underline{\tilde{E}_r\, d\tilde{A}_r}} \quad \text{unverändert!!}$$

$$\boxed{\Phi_E = \int_{\text{Fläche}} \vec{E}\, d\vec{A} = \frac{q_{\text{innen}}}{\varepsilon_0}} = \frac{\sum_i q_i}{\varepsilon_0}$$

$$\Phi_E = \frac{1}{\varepsilon_0} \int \rho \, dV$$

Volumen innerhalb Fläche

Ladungsdichte



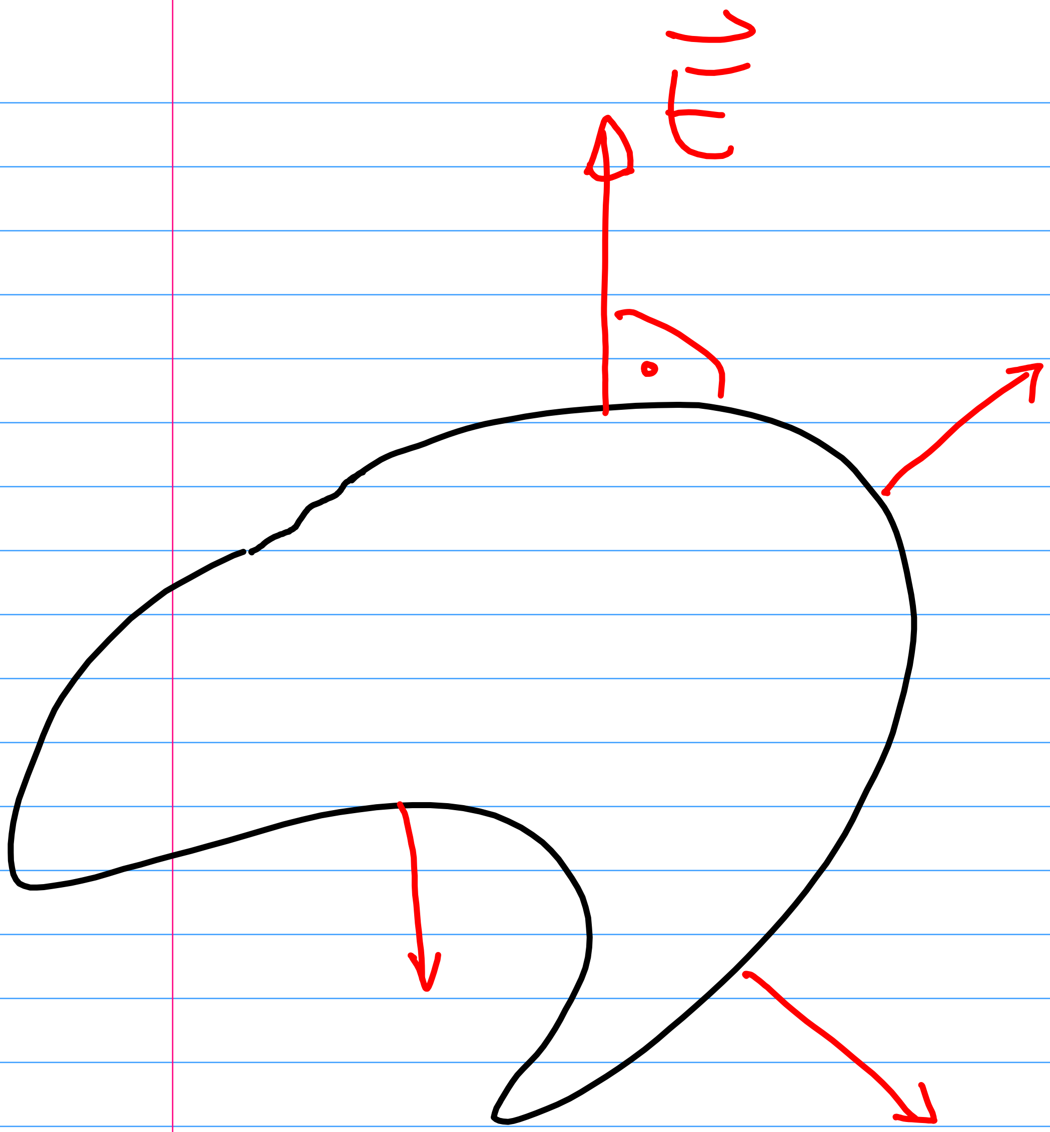
$E = 0$

$$\oint \vec{E}\, d\vec{A} = 0 = \frac{q}{\varepsilon_0}$$

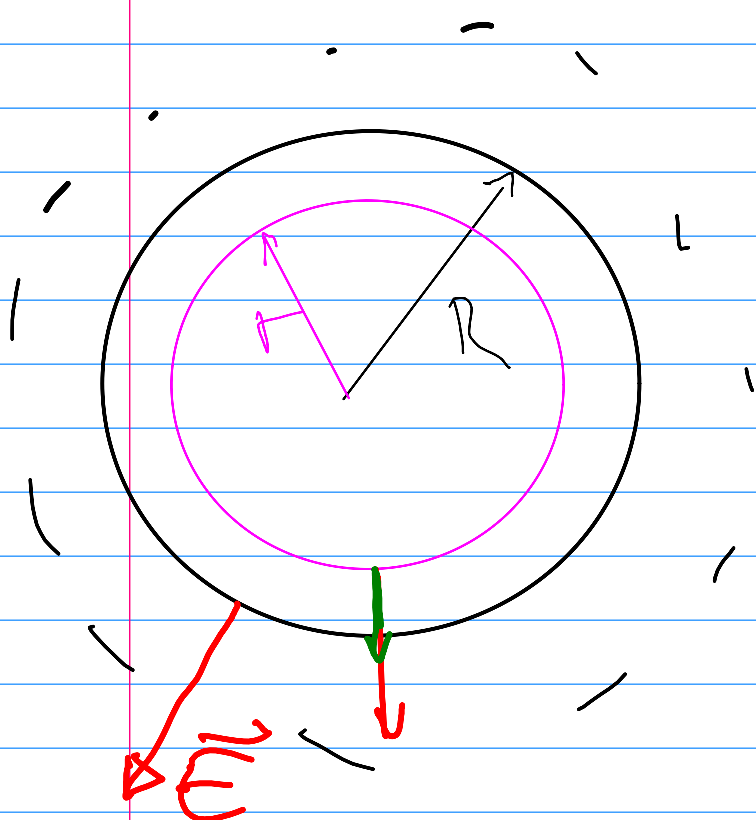
$\Rightarrow q = 0$

$\Rightarrow$ Ladungen an Oberfläche

$\vec{E} \perp$ Oberfläche


$\vec{E}$

Homogen geladene Kugel



$r > R$: $Q_{inner} = q$

$$\int \vec{E}\, d\vec{A} = \frac{q}{\varepsilon_0} = E(r)\, 4\pi r^2$$

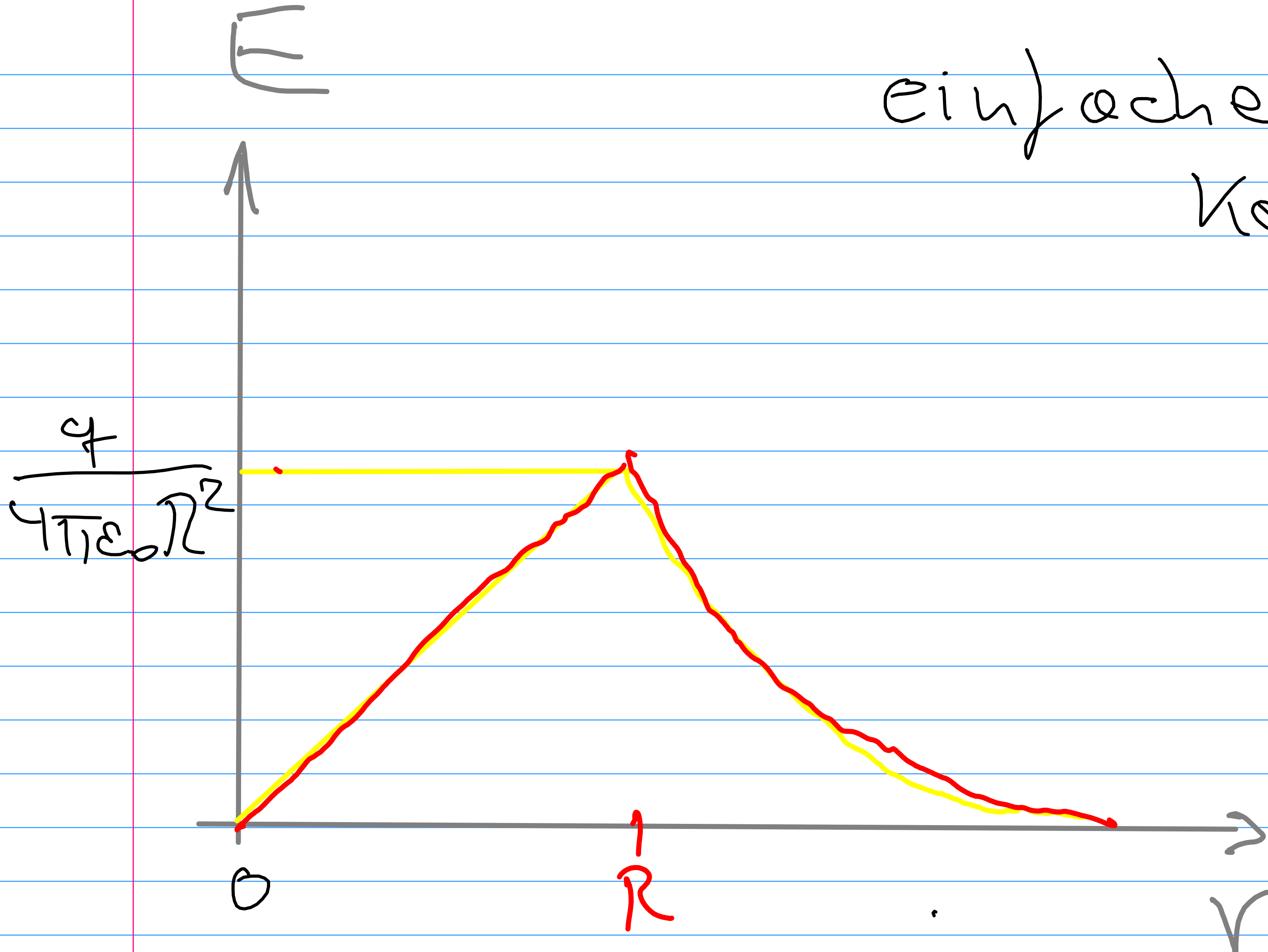
$$\Longrightarrow E(r) = \frac{q}{4\pi\varepsilon_0 r^2}$$

$r < R$:

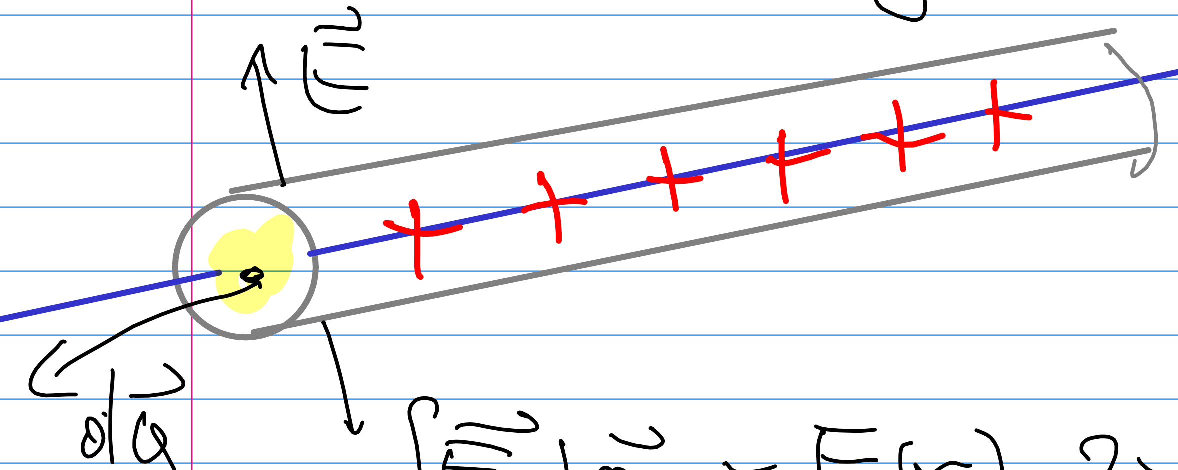
$$Q_{innen} = q \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3$$

$$E(r) = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3}$$

einfaches Kernmodell



(Unendlich) langer Draht



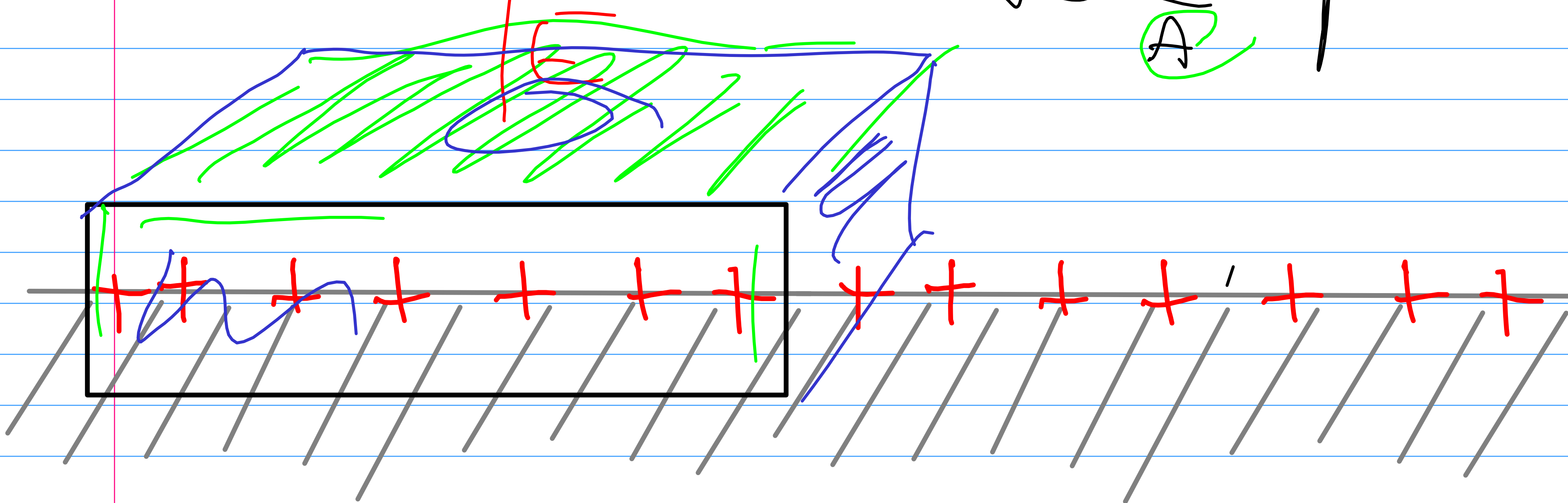
$$Q = \lambda \cdot l$$

$$\int \vec{E}\, d\vec{A} = E(r) \cdot 2\pi r l = \lambda l$$

$$\Rightarrow E(r) = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

Leiteroberfläche

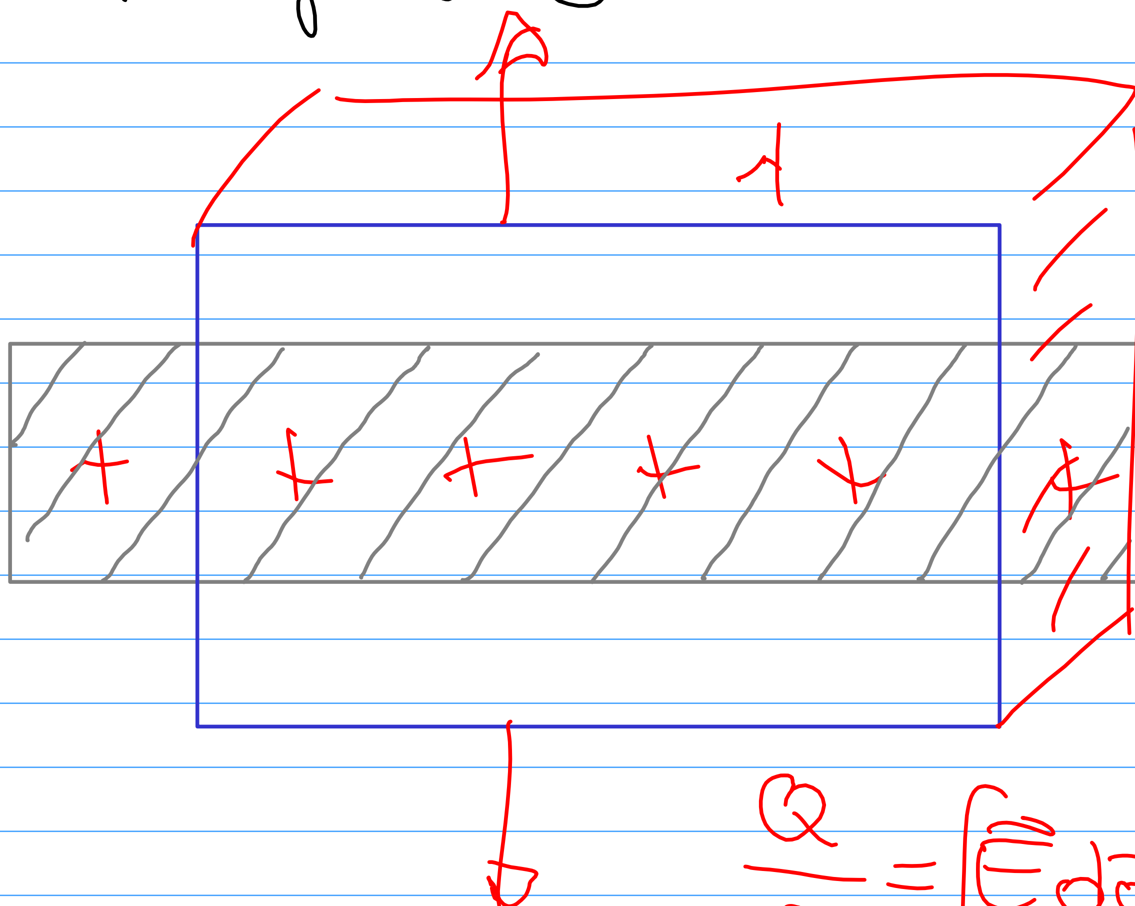
$\sigma = \frac{Q}{A}$



$$\int \vec{E}\, d\vec{a} = E \cdot A = \frac{1}{\varepsilon_0} Q = \frac{1}{\varepsilon_0} \sigma A$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

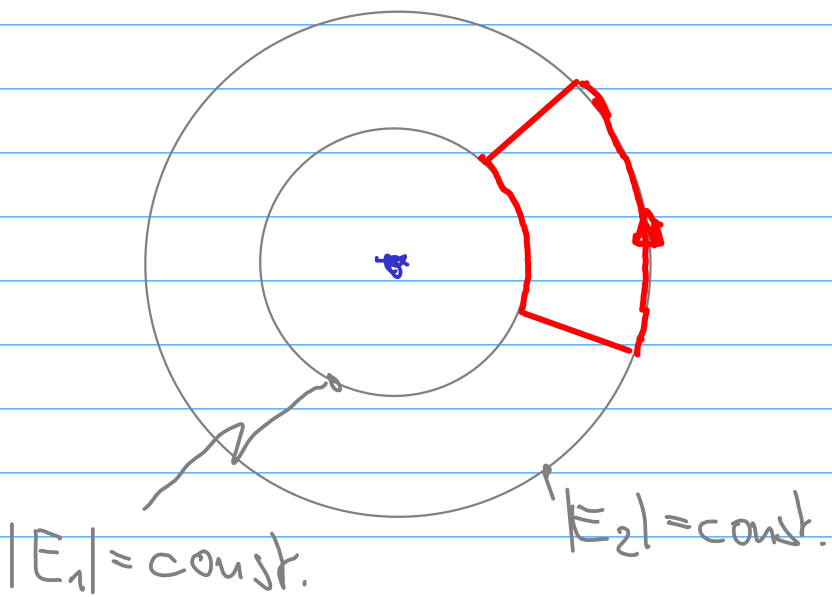
Isolatorplatte



$$\frac{Q}{\varepsilon_0} = \int \vec{E}\, d\vec{a} = E \cdot 2A$$

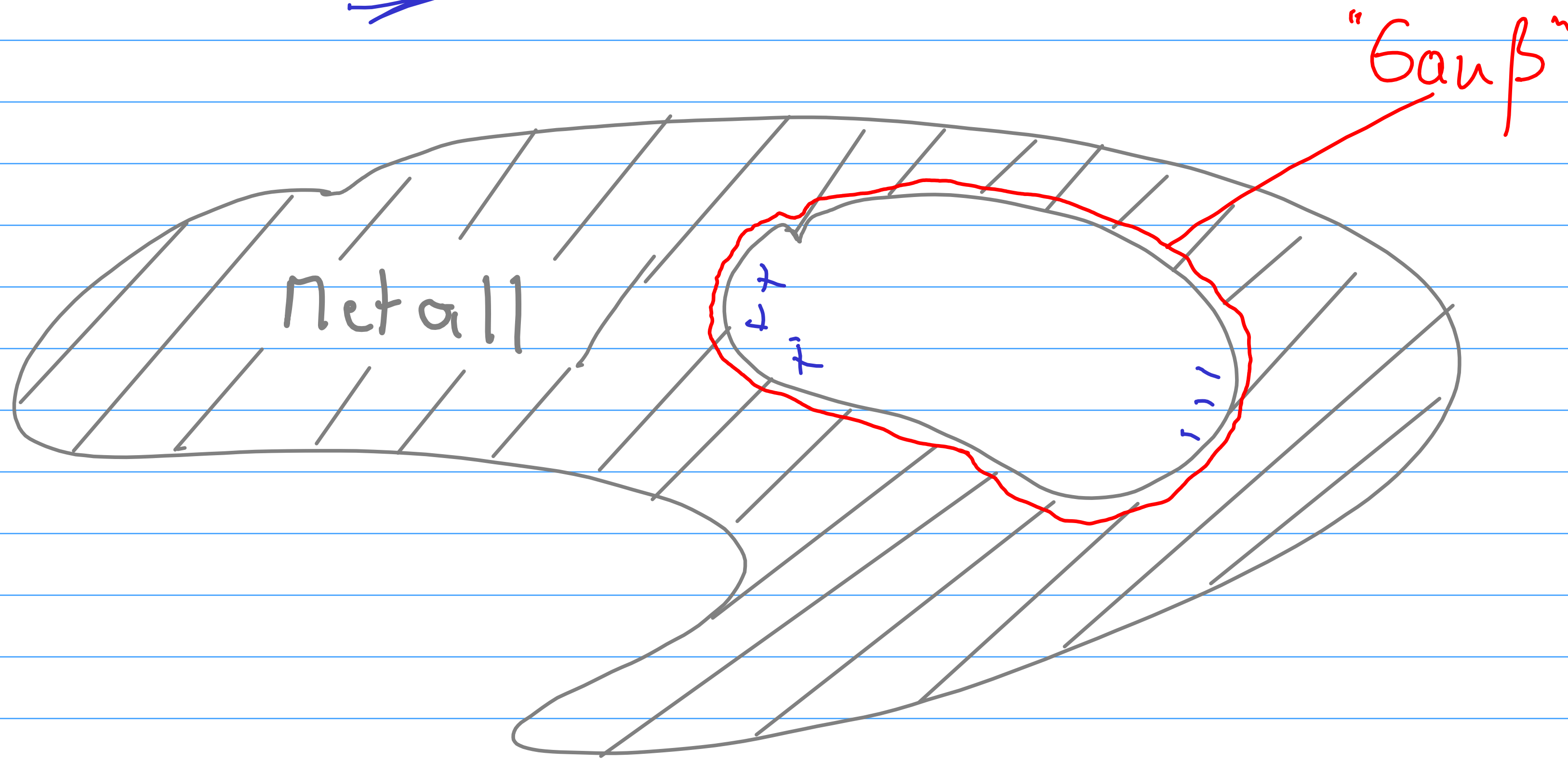
$$\Rightarrow E = \frac{1}{2}\, \sigma / \varepsilon_0$$

Das $\vec{E}$-Feld ist wirbelfrei

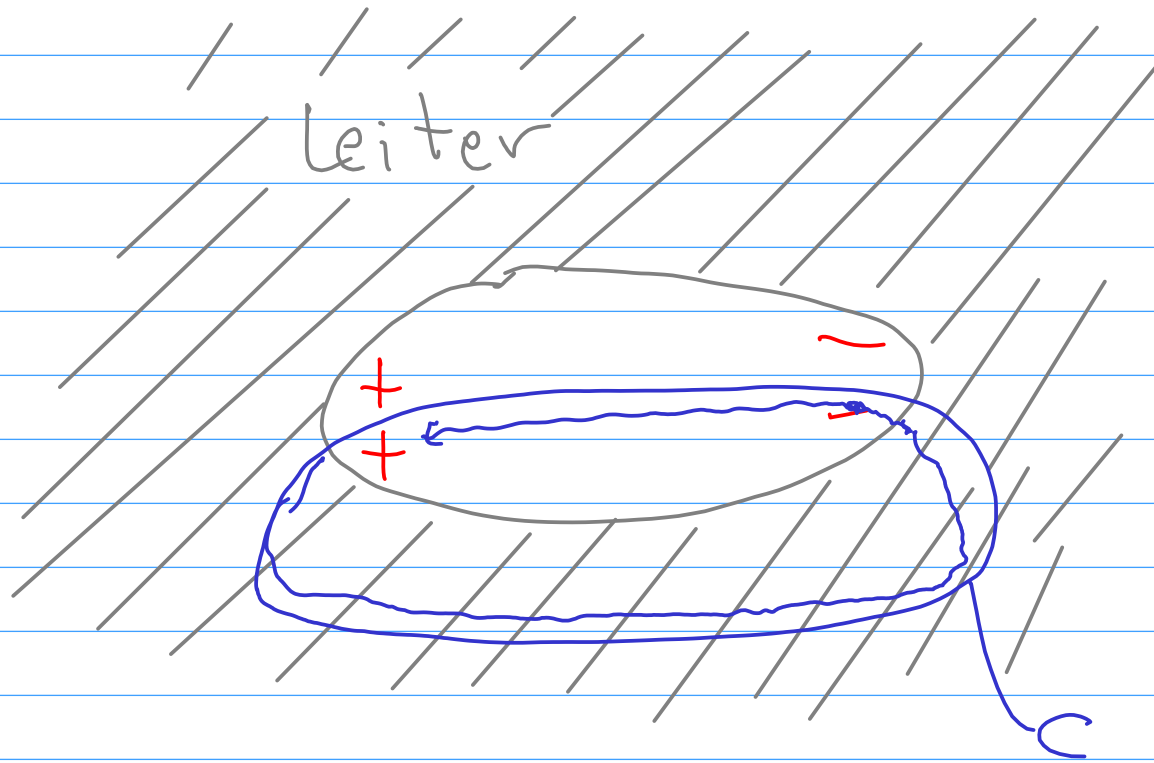


Faradaykäfig via Gauß-Satz

$$\frac{1}{\varepsilon_0} Q = \int\!\!\!\!\int \vec{E}\, d\vec{a} = 0$$



Das gäbe auch $\oint \vec{E}\, d\vec{A} = 0$ ... wirbelfrei konservativ

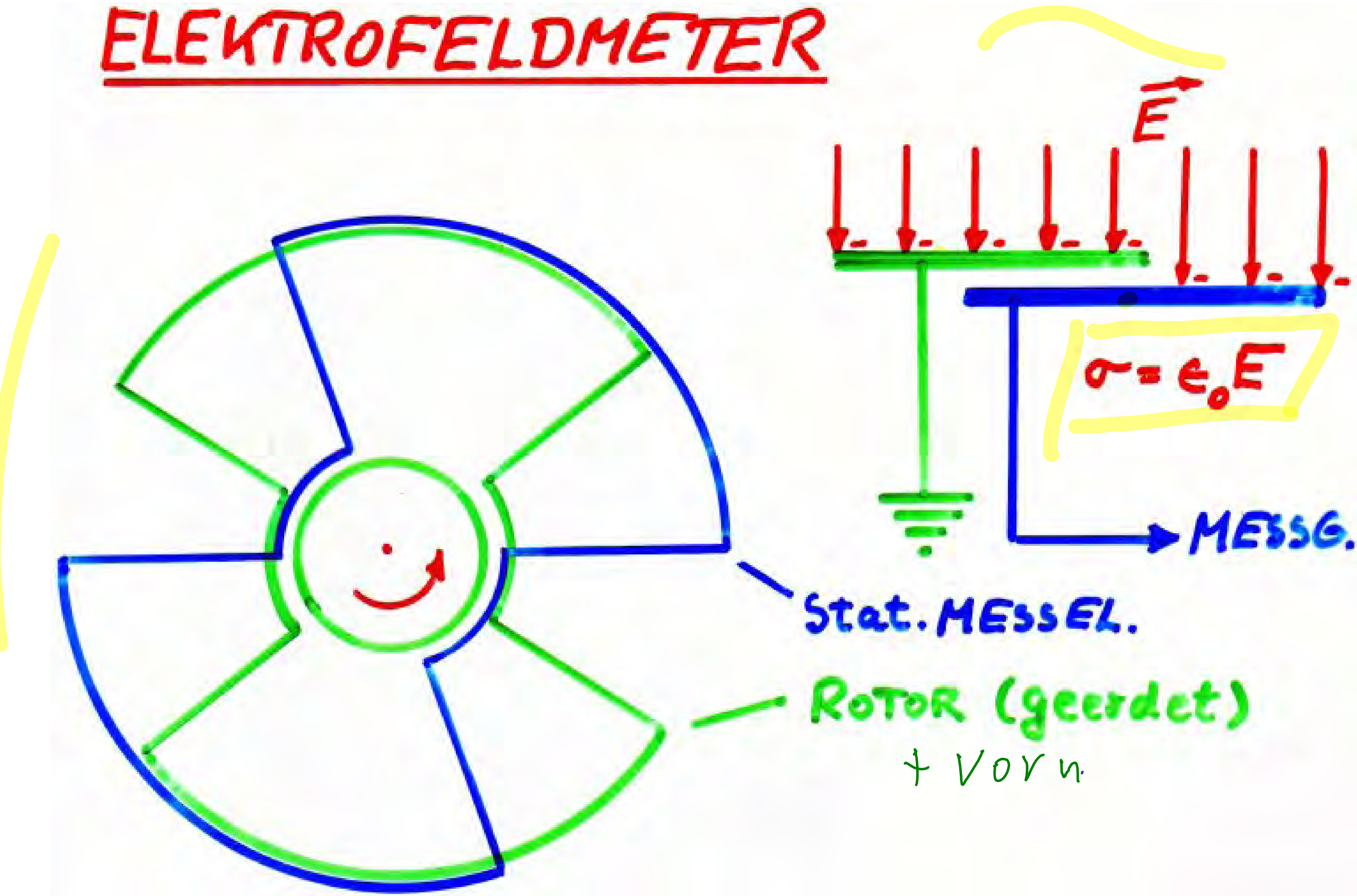


$$\oint_C \vec{E}\, d\vec{r} = 0$$

$\Rightarrow$ Hohlraum in Leiter ist ladungsfrei

$E(r)$

Dipol	$\sim r^{-3}$	(''weit weg'' oder Punktdipol $_{\text{spin}}$)
Punkt	$\sim r^{-2}$	(Punkt ϵ Mathe)
Linie	$\sim r^{-1}$	(1d unendlich oder ''nah d'ran'')
Platte	$\sim r^{0}$ = konstant	(2d unendlich oder ''nah d'ran'')

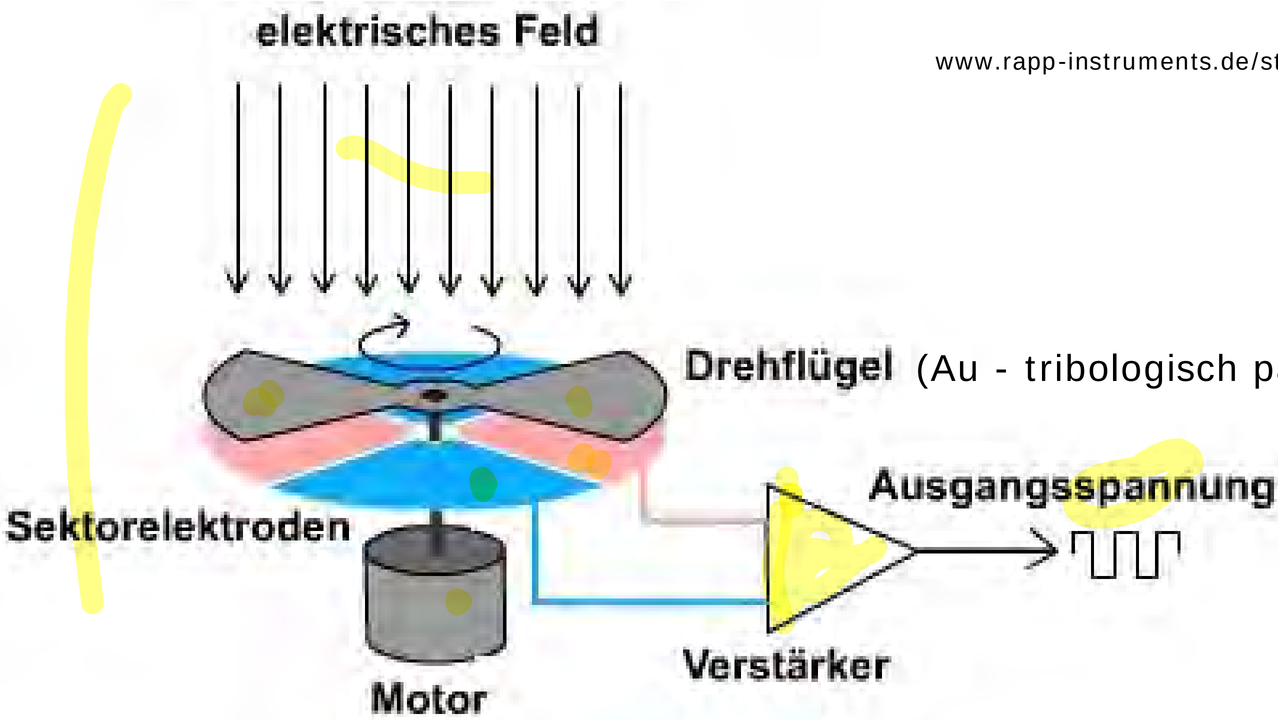


Nutzt Influenz

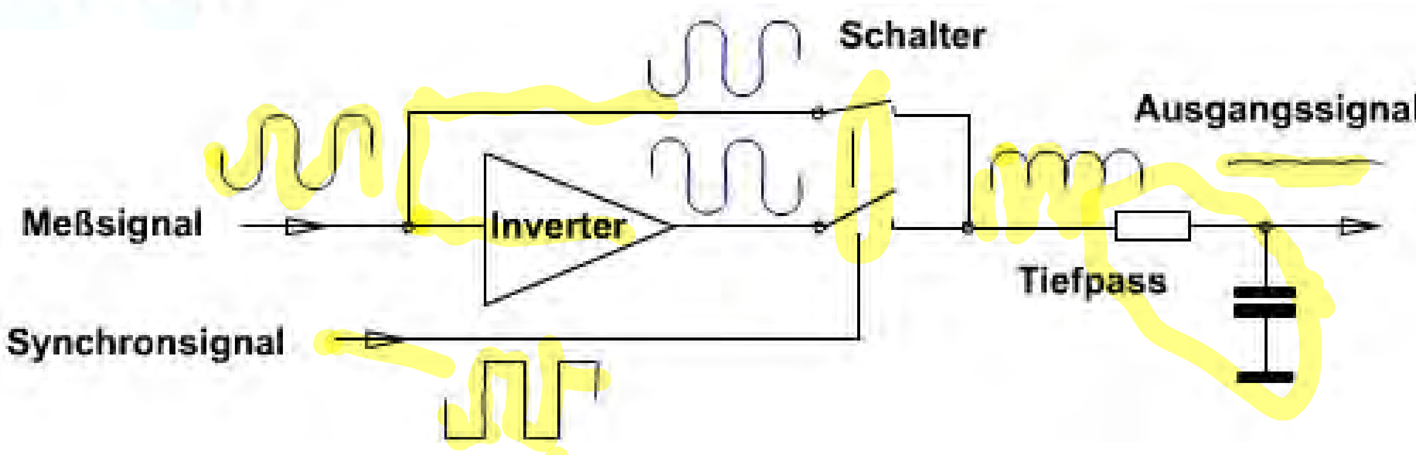
extrem hochohmig

Elektrofeldmeter, Feldmühle

www.rapp-instruments.de/static-machines/measuring/feldmuehle/feldmuehle.htm



(Au - tribologisch passend zu Luft)

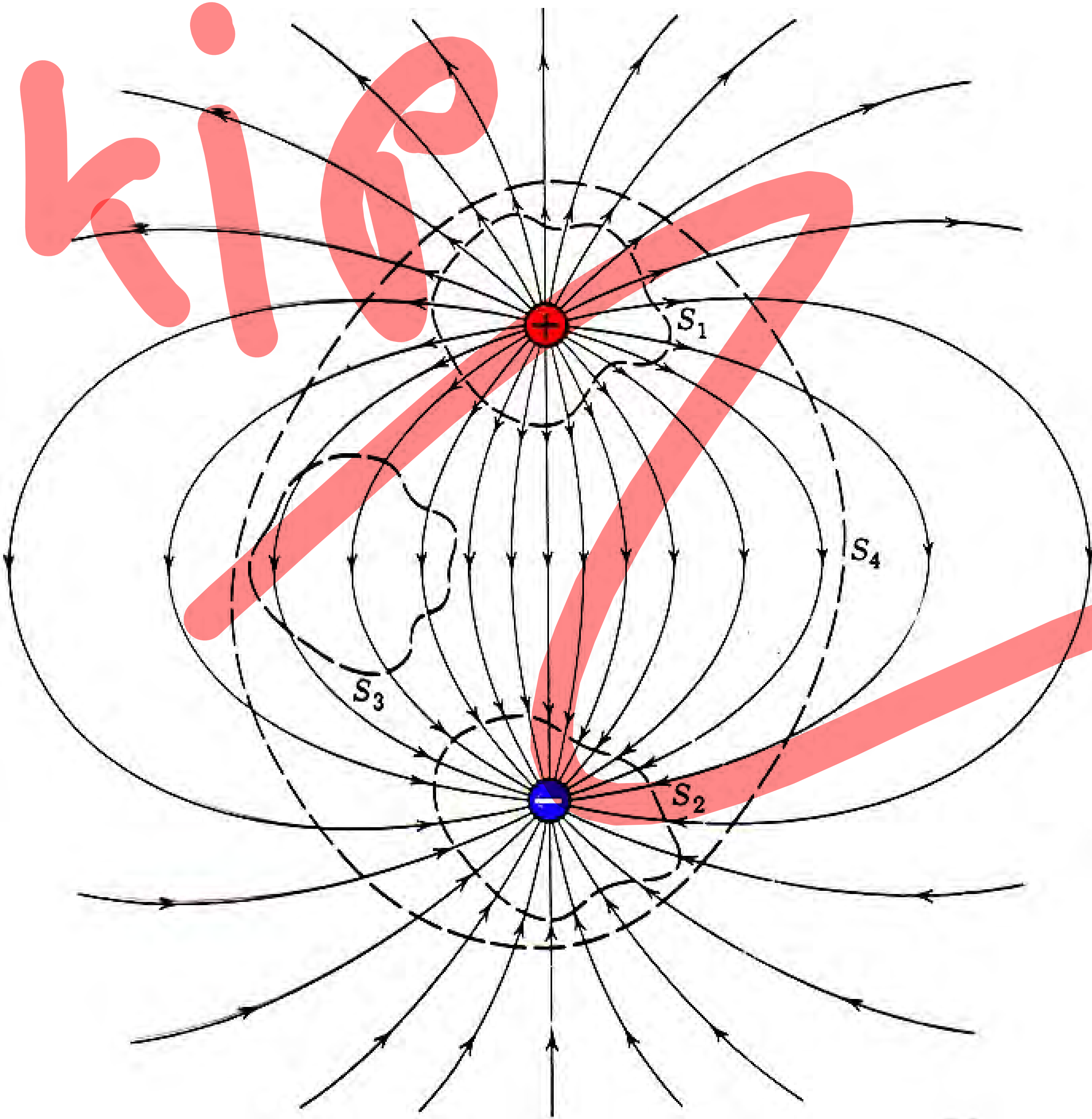


Haupteinsatzgebiet: Meteorologie

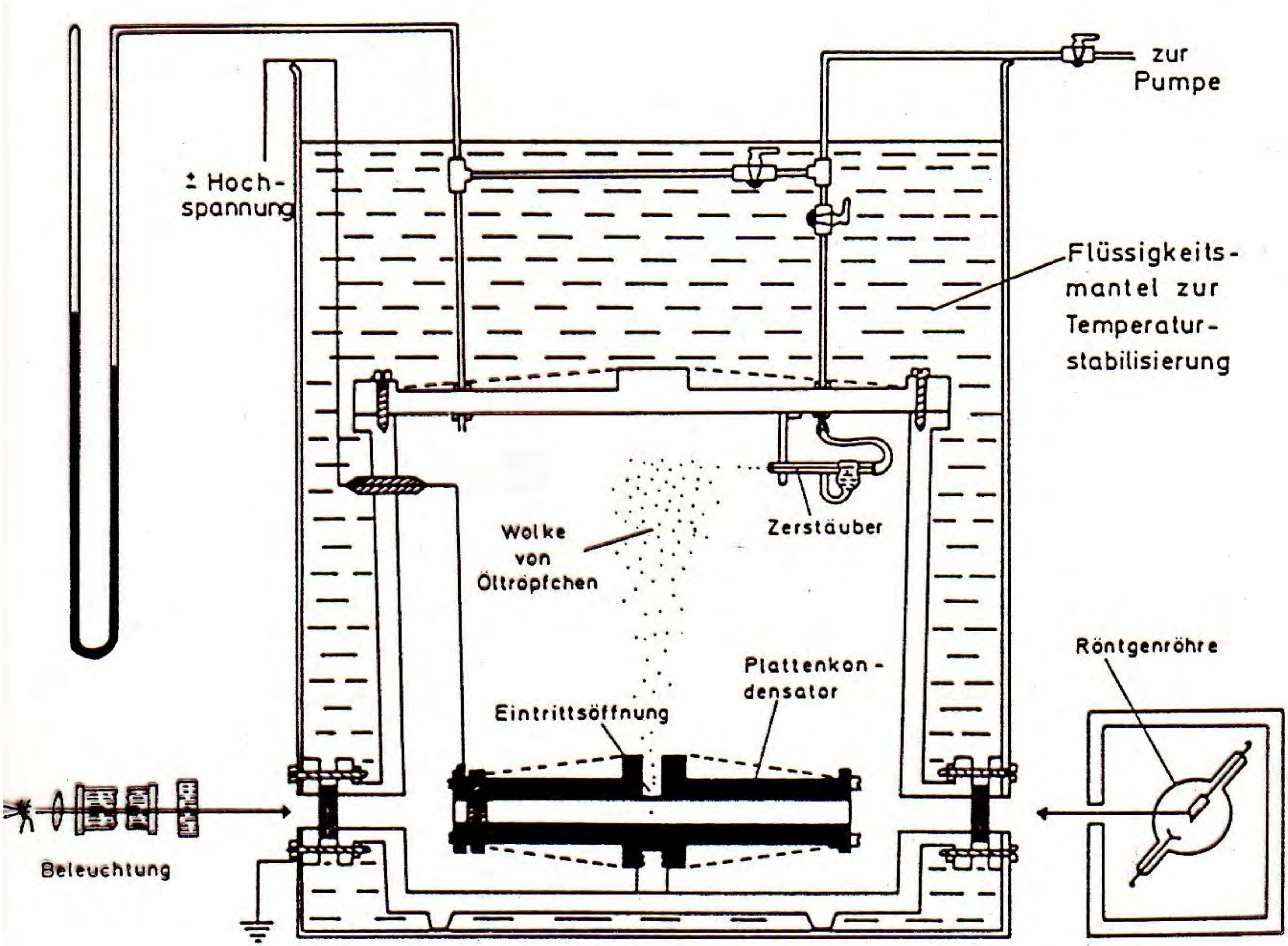
Messung von Veränderungen des elektrischen Feldes in der Erdatmosphäre

(Gewitter, Kalt-/Warmfrontdurchgänge, Regenwolken ...)

Skip

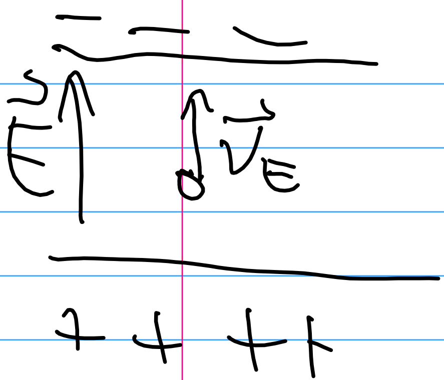


Millikan 1911



$e = 1{,}60210 \cdot 10^{-19}\ C$

Glühlampe $\approx 10^{18}\ \frac{e}{s}$



ohne E-Feld:

$$-\frac{4}{3}\pi r^3(\rho-\rho_{Luft})g = 6\pi\eta r v_g \quad (*)$$

Tröpfchenradius

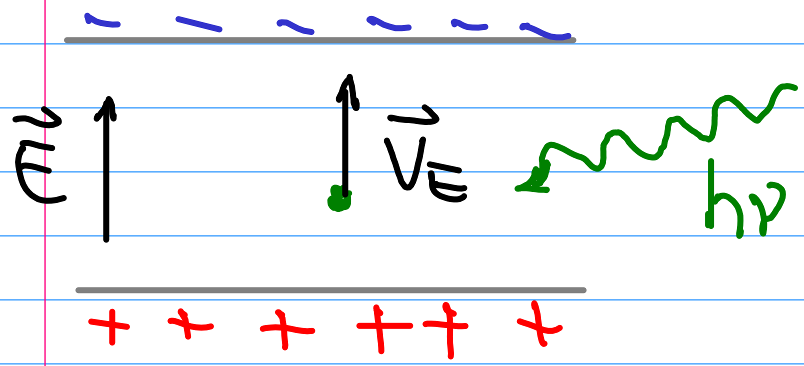
Öl-dichte

Geschw. ohne E-Feld

mit E-Feld:

$$q\cdot E-\frac{4}{3}\pi r^3(\rho-\rho_{Luft})g = 6\pi\eta r v_E \quad (**)$$

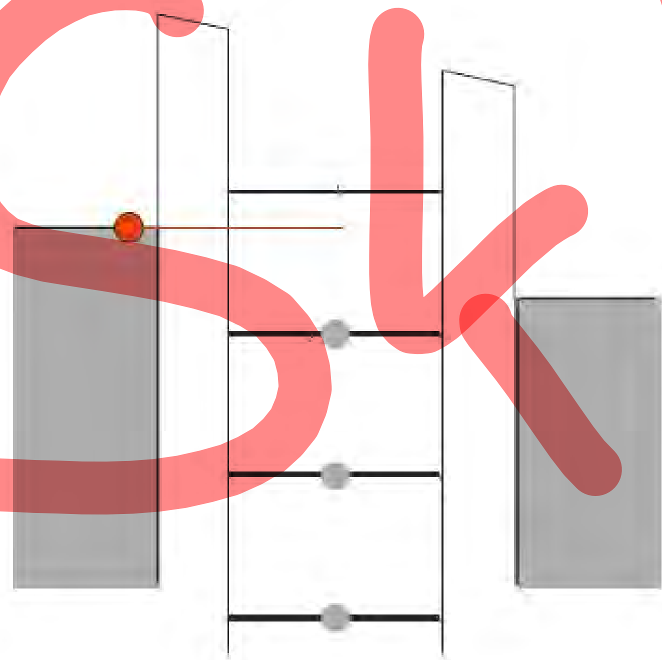
$$\Rightarrow qE = 6\pi\eta r(v_E-v_g) \quad (***)$$

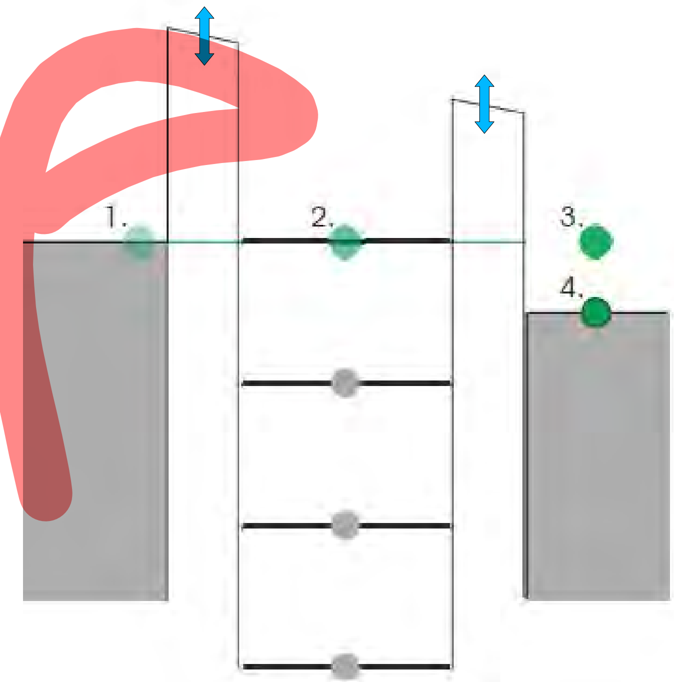


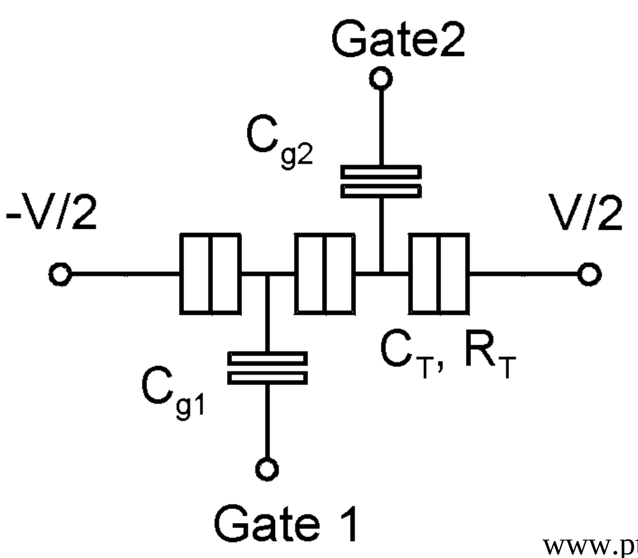
$\vec{E}$
$\vec{V_E}$
$h\nu$

Elektronendrehtür (turnstile)

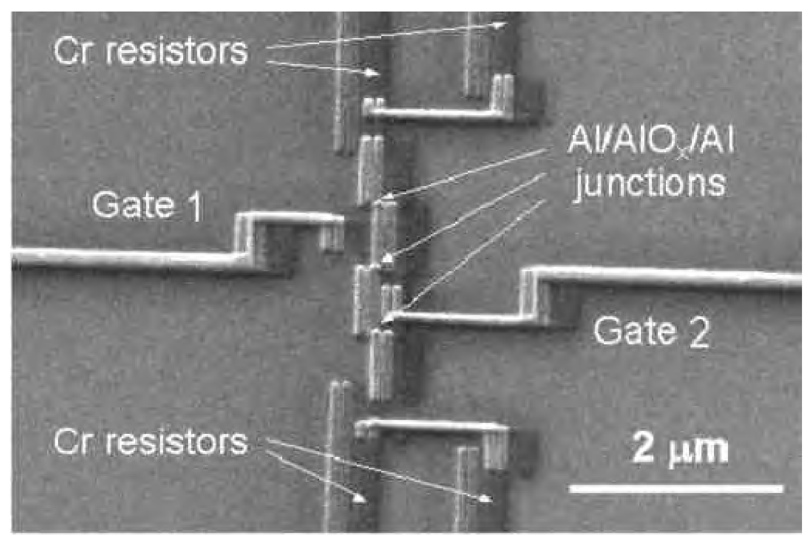
Skip



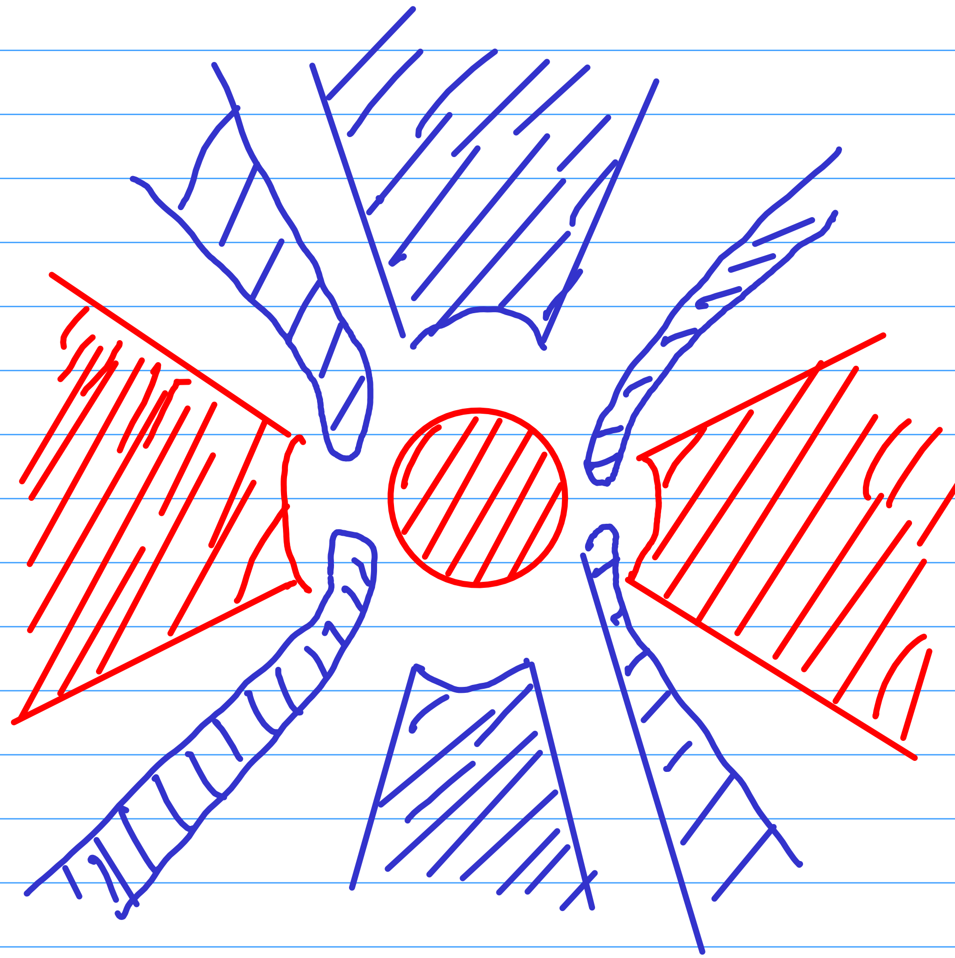




www.ptb.de



Aufsicht



Skip

v_{g}, v_E; $(*)$ nach r auflösen

$r \to (***)$ einsetzen

$$q = 9\sqrt{2}\,(v_E - v_g)\sqrt{v_g}\,\eta^{3/2}\,\frac{1}{E(\rho_{Luft} - \rho)\sqrt{g}}$$

$$\Longrightarrow \quad q = n \cdot e\,;\; n \in \mathbb{N}_0$$