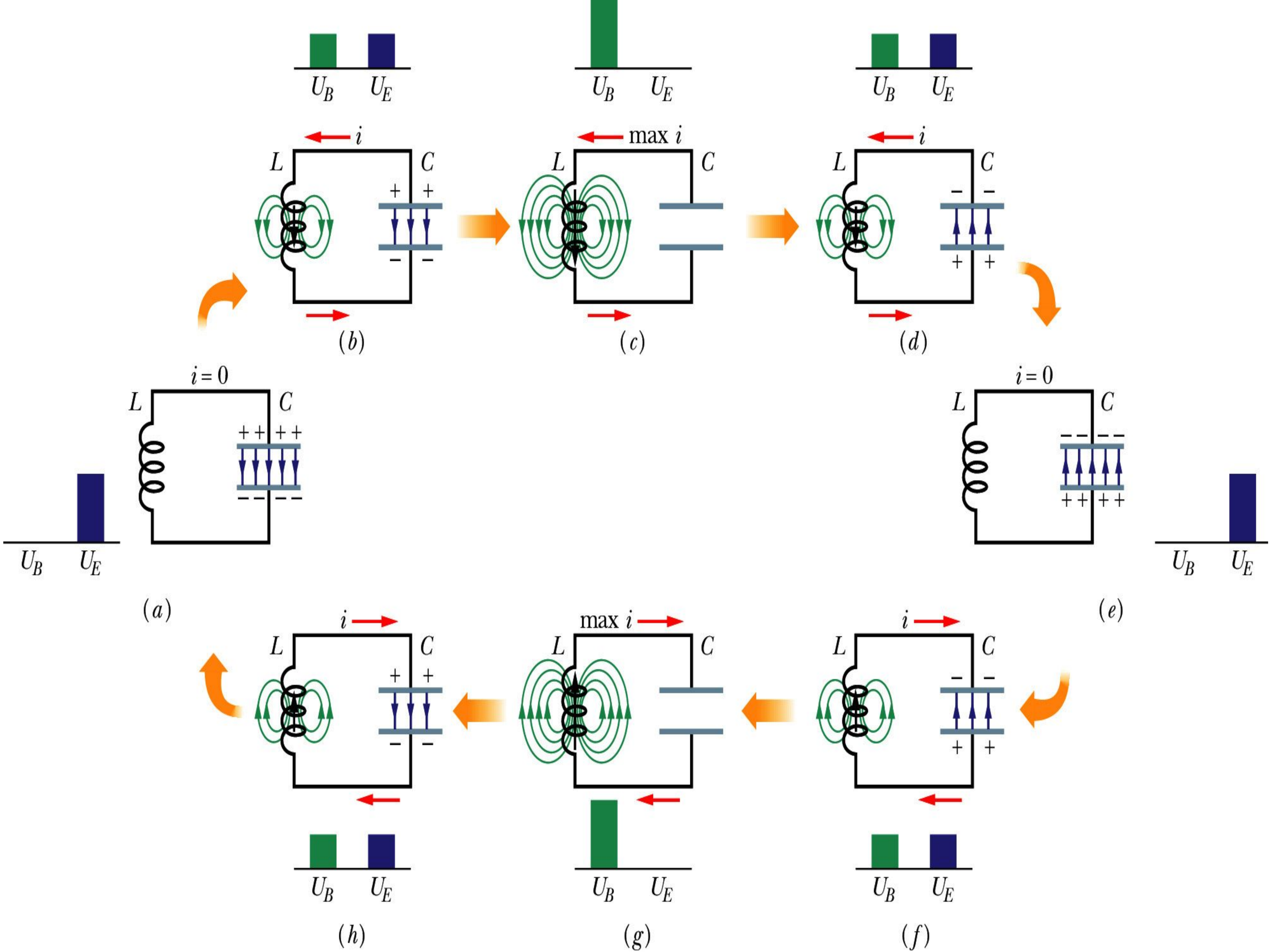


U_B U_E
i
L C
(b)
max i
(c)
(d)
$i = 0$
(a)
(e)
(h)
(g)
(f)

Elektromech. Analogie

Feder $\frac{1}{2} D x^2$	Kond. $\frac{1}{2} \frac{1}{C} Q^2$
Masse $\frac{1}{2} m v^2$	Spule $\frac{1}{2} L I^2$
$v = \dot{x}$	$I = \dot{Q}$
$\omega_0 = \sqrt{D/m}$	$\omega_0 = \frac{1}{\sqrt{LC}}$

LC-Kreis (ohne R)

$$W = W_B + W_E = \frac{1}{2} L I^2 + \frac{1}{2} \frac{1}{C} Q^2$$

$$R = 0 \curvearrowright \dot{W} = 0$$

$$\frac{d}{dt}\left(\frac{L I^2}{2} + \frac{Q^2}{2C} \right) = 0 = L I \dot{I} + \frac{1}{C} Q \dot{Q}$$

$$= L \dot{Q} \ddot{Q} + \frac{1}{C} Q \dot{Q}$$

$$\Rightarrow L \ddot{Q} + \frac{1}{C} Q = 0$$

Allgm. Lsg.: $Q(t) = Q_0 \cos(\omega_0 t + \varphi)$; $\omega_0 = \frac{1}{\sqrt{LC}}$

$$I(t) = -\underbrace{Q_0 \omega_0}_{} \sin(\omega_0 t + \varphi)$$

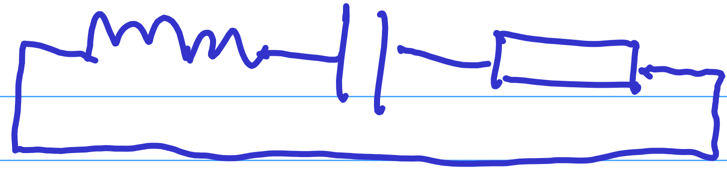
$$= -I_0 \sin(\omega_0 t + \varphi)$$

$$W_E = \frac{1}{2C} Q^2 = \frac{Q_0^2}{2C} \cos^2(\omega_0 t + \varphi)$$

$$W_B = \frac{1}{2} L I^2 = \frac{1}{2} L \omega_0^2 Q_0^2 \sin^2(\omega_0 t + \varphi)$$

$$= \frac{Q_0^2}{2C} \sin^2(\omega_0 t + \varphi)$$

RLC-Kreis



$$\frac{d}{dt}\left(W_B + W_E\right) = -I^2 R \qquad P = -I^2 R$$

$$\Rightarrow L I \dot{I} + \frac{1}{C} Q \dot{Q} = -I^2 R$$

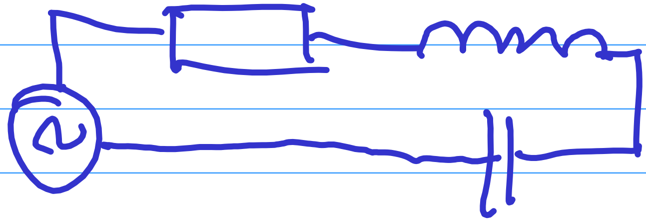
$$\Rightarrow L \ddot{Q} + \frac{1}{C} Q + R \dot{Q} = 0$$

$$\Rightarrow Q(t) = Q_0 \exp\left(-\frac{1}{2}\frac{R}{L} t\right) \cos(\omega' t + \varphi)$$

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{1}{2}\frac{R}{L}\right)^2} \; ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Gedämpfte Schwing. mit Ampl. $Q_0 e^{-\frac{1}{2}\frac{R}{L}t}$

RLC-Reihenschwingkreis mit ext. Anregung



U

$$U_R + U_L + U_C = U$$

$$RI + L\dot{I} + \frac{Q}{C} = \hat{u}\, e^{i\omega t} \qquad \Big| \, d/dt$$

$$L\ddot{I} + R\dot{I} + \frac{1}{C} I = \hat{u}\, i\omega\, e^{i\omega t}$$

$$\ddot{I} + 2\delta \dot{I} + \omega_0^2 I = \frac{\omega}{L} \hat{u}\, e^{i(\omega t + \pi/2)}$$

$$\delta = \frac{1}{2}\frac{R}{L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Stationäre Lösung $\left(t \gg \frac{1}{\delta}\right)$

oszilliert mit ω, φ möglich

Ansatz: $I = \hat{I}\, e^{i(\omega t - \varphi)}$

$$\Rightarrow -\omega^2 I + 2\delta i\omega I + \omega_0^2 I = \frac{\omega}{L}\hat{U}\, e^{i(\omega t + \pi/2)} \quad (*)$$

Teilen durch $\hat{I}\, e^{i\omega t}$

$$\Rightarrow -\omega^2 e^{-i\varphi} + 2i\omega\delta e^{-i\varphi} + \omega_0^2 e^{-i\varphi} =$$

$$\ldots = \frac{\omega}{L}\frac{\hat{U}}{\hat{I}}\, e^{i\pi/2}$$

Realteil: $-\omega^2 \cos\varphi + 2\delta\omega \sin\varphi + \omega_0^2 \cos\varphi = 0$

$$\frac{\sin\varphi}{\cos\varphi} = \tan\varphi = \frac{\omega_0^2 - \omega^2}{2\delta\omega}$$

$\omega < \omega_0 \Rightarrow$ rechts $> 0 \longrightarrow 90°$

$\omega > \omega_0$ $\quad < 0 \longrightarrow -90°$

$\omega = \omega_0$ $\quad 0°$

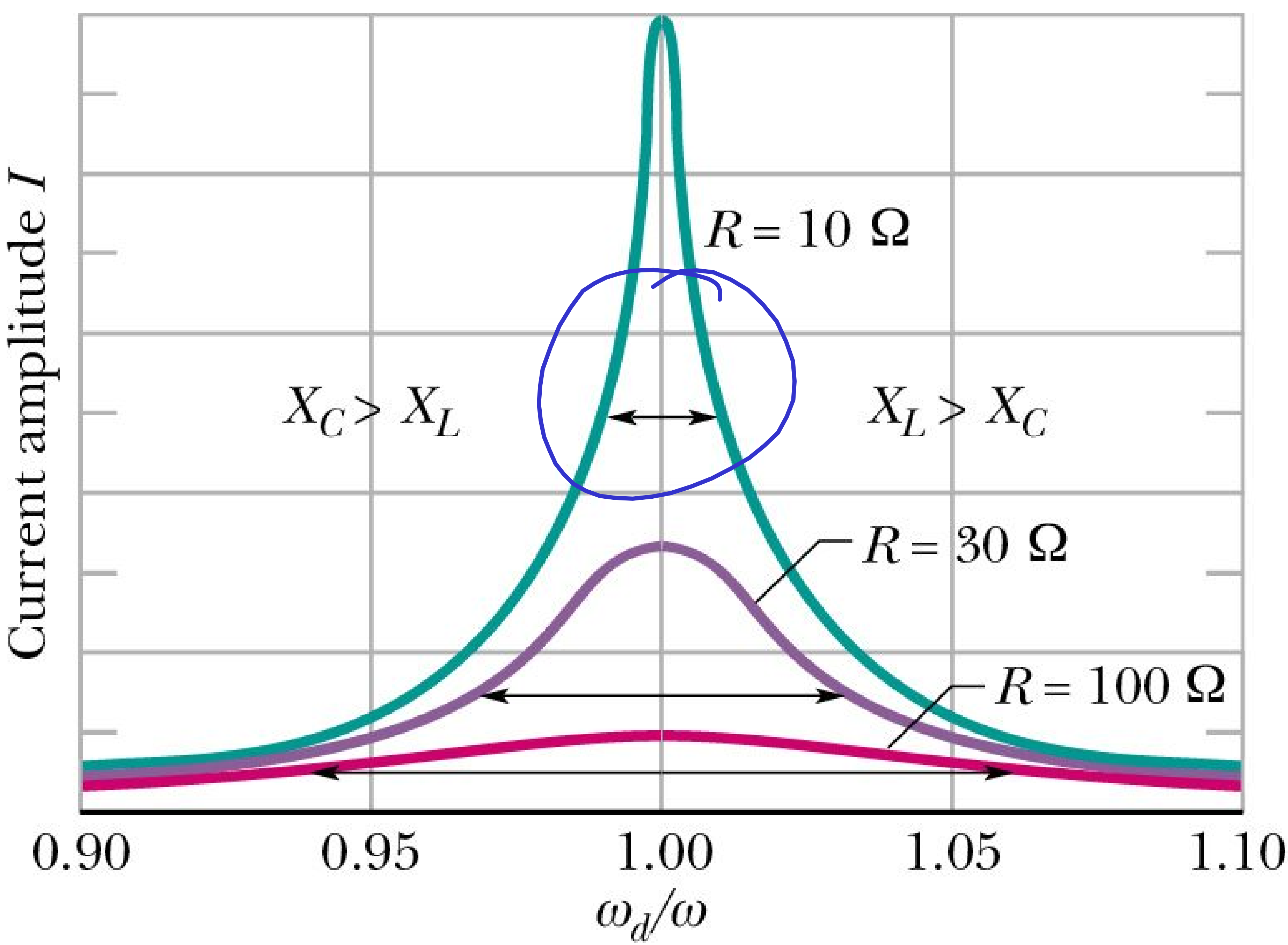
Nimm (*), sortiere nach I:

$$I = \hat{I} e^{i(\omega t - \varphi)} = \frac{\frac{\omega}{L}\hat{u} e^{i(\omega t + \frac{\pi}{2})}}{\omega_0^2 - \omega^2 + 2\delta i \omega}$$

Betrag:

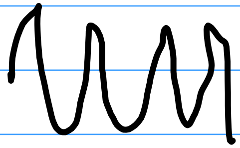
$$\hat{I} \, 1 = \frac{\frac{\omega}{L}\hat{u} \, 1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$$

$$\Rightarrow \hat{I} = \hat{u} \frac{\omega}{L} \frac{1}{\sqrt{\ldots}}$$



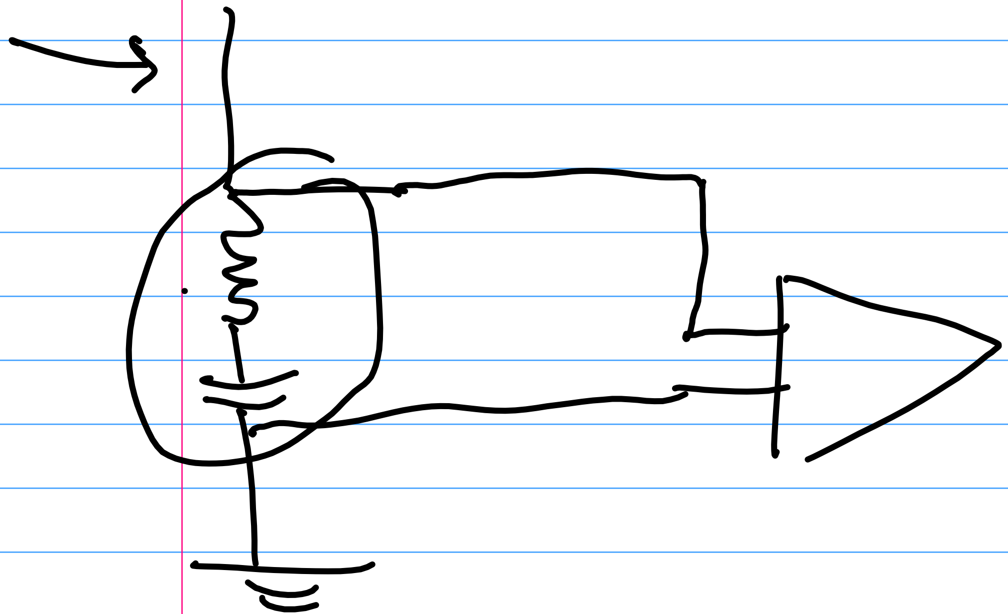
Current amplitude I
$R = 10\ \Omega$
$R = 30\ \Omega$
$R = 100\ \Omega$
$X_C > X_L$
$X_L > X_C$
0.90
0.95
1.00
1.05
1.10
ω_d/ω

Sender: Trägerfrequenz



100 kHz → 100 MHz

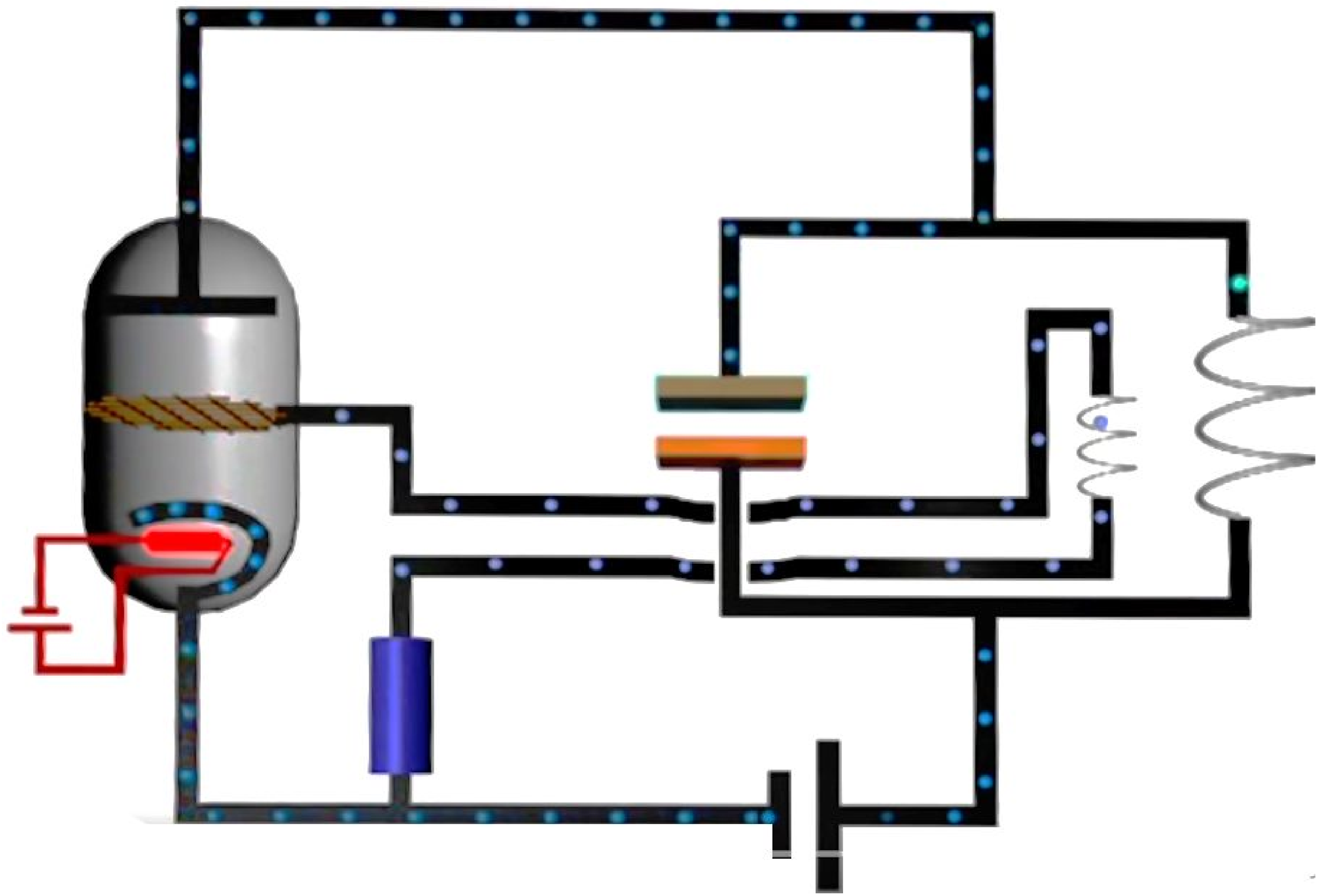
AM



p.s.

mehr finden Sie unter den Stichwörtern "Amplitudenmodulation" und "Frequenzmodulation" https://www.youtube.com/watch?v=4XRnWXcIDOk

Meißnerschaltung



Induktive Kopplung

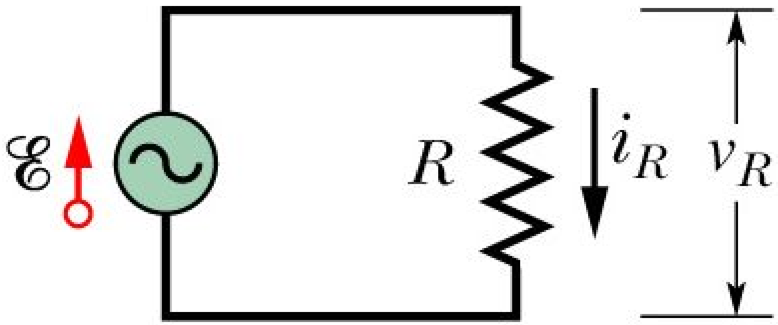
Rückkopplung: entdämpfter Schwingkreis

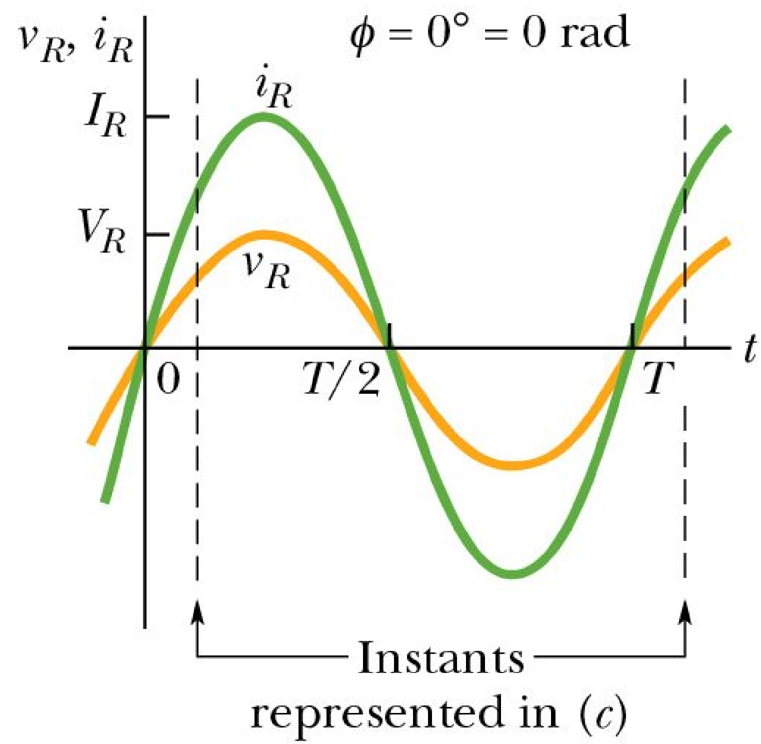
Komplexe Widerstände
(Wechselstromlehre)

R

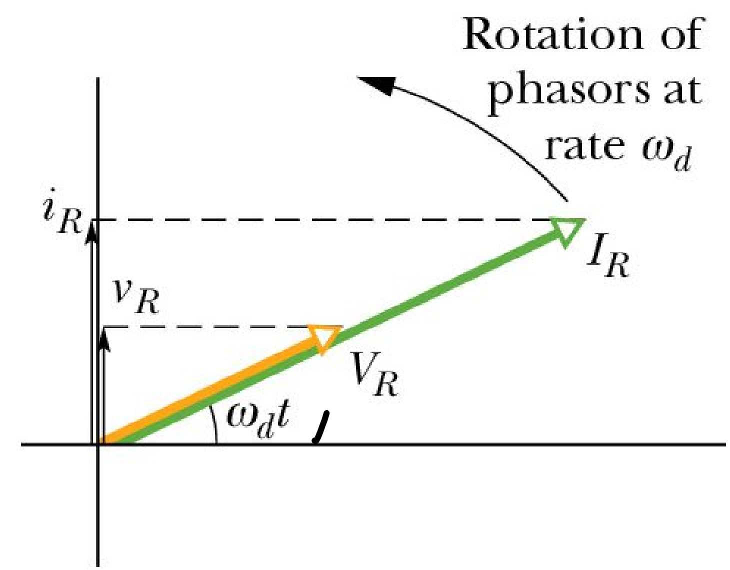
$$U_R = U_{R,max} \cdot \sin(\omega t)$$

$$I_R = \frac{U_R}{R} = \frac{U_{R,max}}{R} \sin(\omega t)$$





(b)



Halliday, 33-8

C

$$u_C = u_{C,max} \sin(\omega t)$$

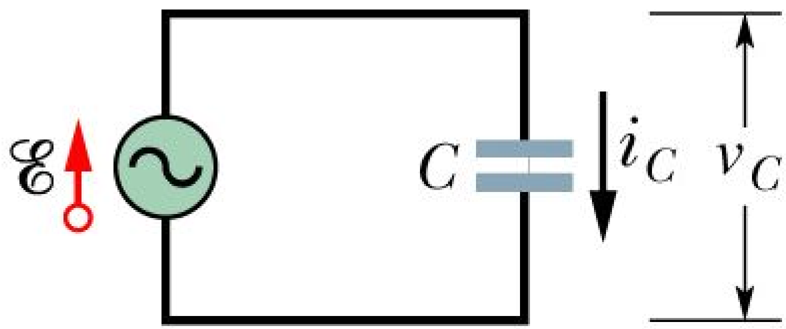
$$Q = C \cdot u_C$$

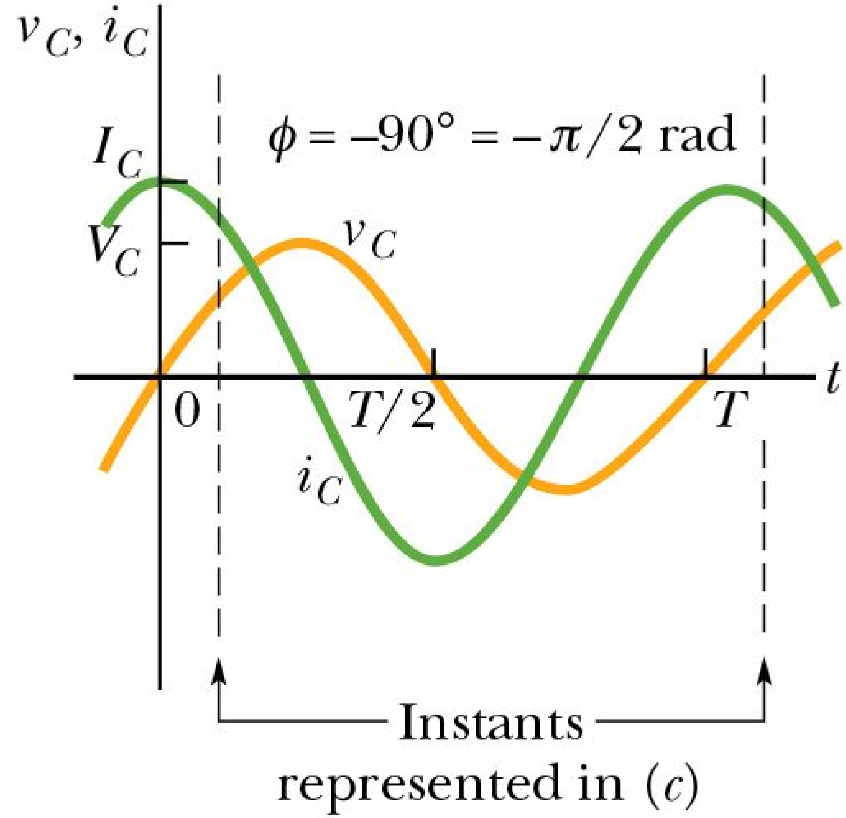
$$I = \dot{Q} = \omega C \, u_{C,max} \cos(\omega t)$$

$$= \frac{u_{C,max}}{X_C} \sin(\omega t + 90^\circ)$$

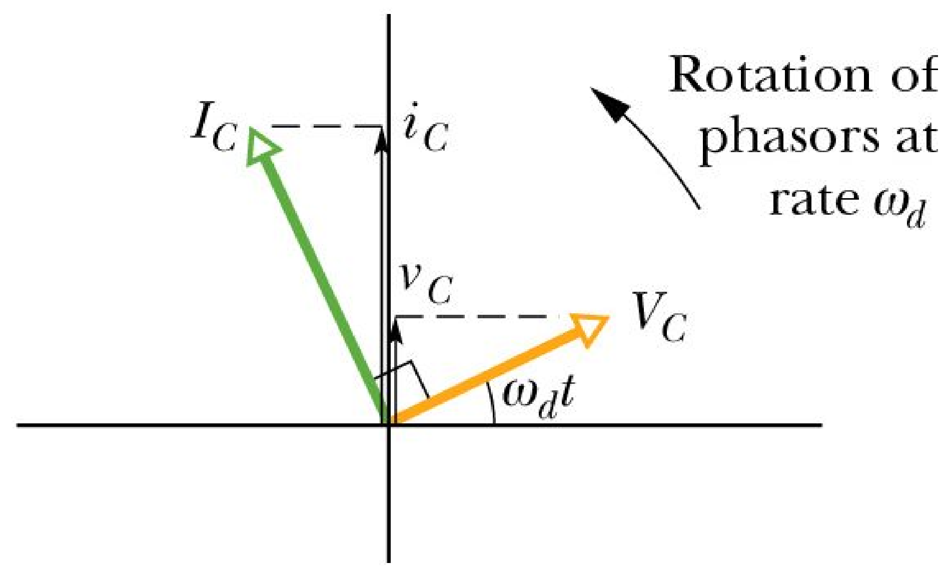
mit $X_C = \dfrac{1}{\omega C}$

(vgl. $I = \dfrac{U}{R}$)





(b)



Halliday, 33-9

L

$$U_L = U_{L\,max} \sin \omega t$$

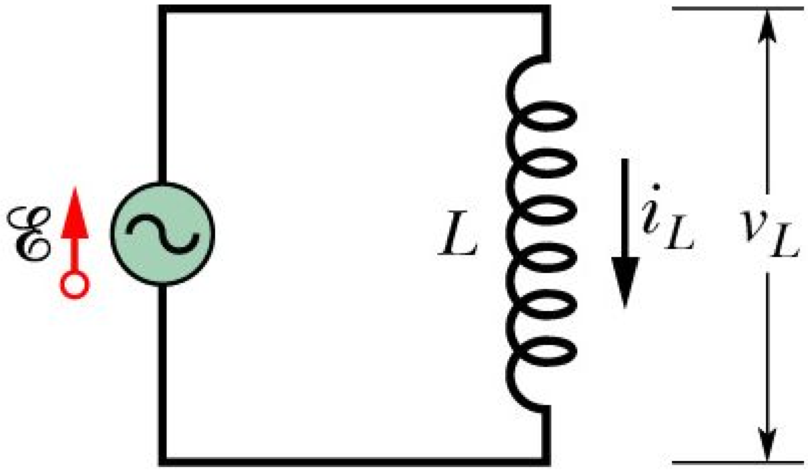
$$U_L = L\dot{i} \Rightarrow \dot{i} = \frac{U_{L,max}}{L} \sin(\omega t)$$

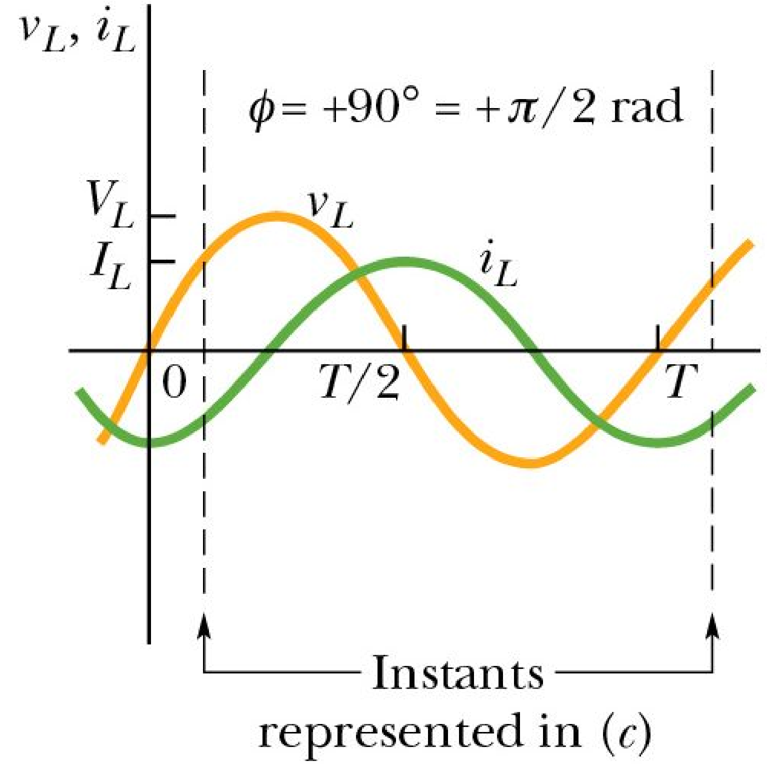
$$I(t) = \int dI = \int dt \frac{U_{L,max}}{L} \sin(\omega t)$$

$$= -\frac{U_{L,max}}{\omega L} \cos(\omega t)$$

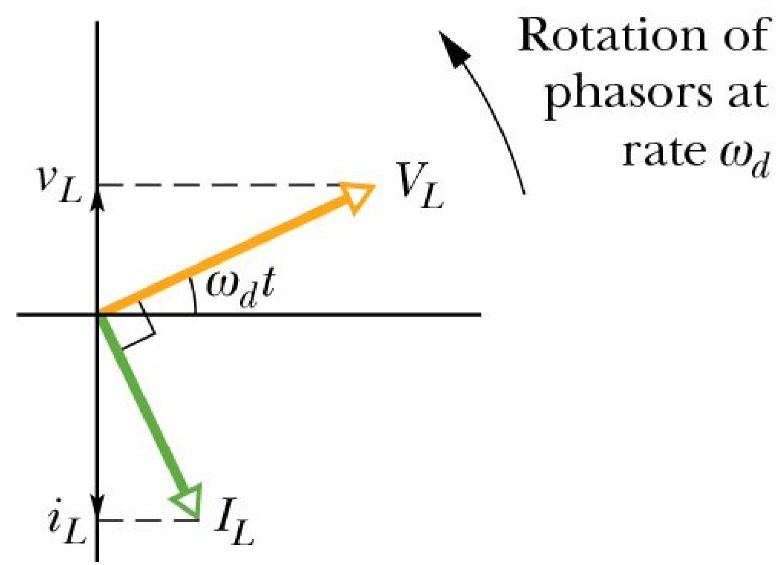
$$= \frac{U_{L\,max}}{\omega L} \sin(\omega t - 90^\circ) = \frac{U_{L,max}}{X_L} \sin(\omega t - 90^\circ)$$

$$X_L = \omega L$$





(b)

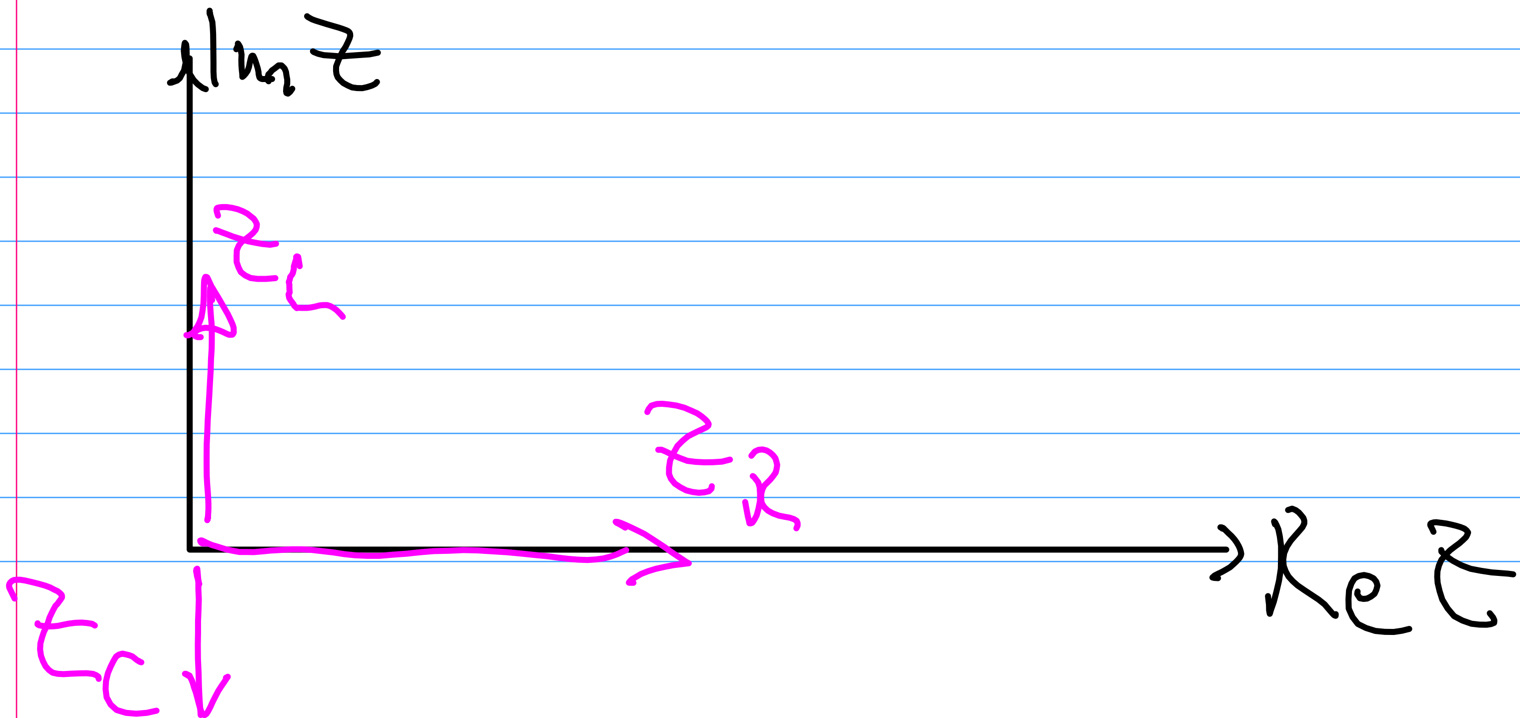


Halliday, 33-10

Zeiger: $\frac{U}{I} = X$, Winkel φ

komplexe Zahl: $Z = X e^{i\varphi}$

$$Z_C = \frac{1}{i\omega C} \qquad Z_L = i\omega L \qquad Z_R = R$$



$$Z = A + i \cdot B$$

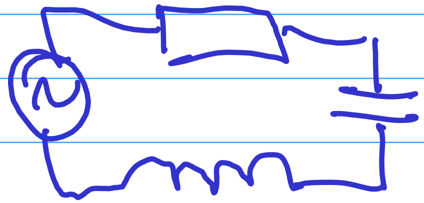
Blindwiderstand

Wirkwiderstand

Kompl. Widerstand   Impedanz

$|Z|$ = Scheinwiderstand

RLC mit kompl. Widerständen



$U = Z I \quad ; \quad Z = Z(R, L, C)$

$$I = \frac{1}{|Z|} e^{-i\varphi} U$$

$$Z = R + i\omega L + \frac{1}{i\omega C}$$

$$|Z| = \sqrt{(\mathrm{Re}\, Z)^2 + (\mathrm{Im}\, Z)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\varphi = \frac{\mathrm{Im}\, Z}{\mathrm{Re}\, Z} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Güte des Resonators – Q-Faktor

$$I(\omega) = \frac{1}{\sqrt{2}} I(\omega_0) \Longleftrightarrow Z(\omega) = \sqrt{2}\, Z(\omega_0)$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\, R \qquad \left(\omega_0 L - \frac{1}{\omega_0 C} = 0\right)$$

$$\Rightarrow R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\Rightarrow \pm R = \omega L - \frac{1}{\omega C}$$

$$\Rightarrow \pm R\omega C = \omega^2 LC - 1$$

$$\Rightarrow \pm \frac{R}{L}\frac{\omega}{\omega_0^2} = \frac{\omega^2}{\omega_0^2} - 1$$

$$\Rightarrow \pm 2\delta\omega = \omega^2 - \omega_0^2$$

$$\Rightarrow \omega = \pm\sqrt{\omega_0^2 + \delta^2} \pm \delta$$

$$\Rightarrow \omega = \pm\omega_0\sqrt{1 + \left(\frac{\delta}{\omega_0}\right)^2} \pm \delta$$

Anm.: $\delta \ll \omega_0$ (Kl. Dämpfung)

$$\Delta\omega = 2\delta$$

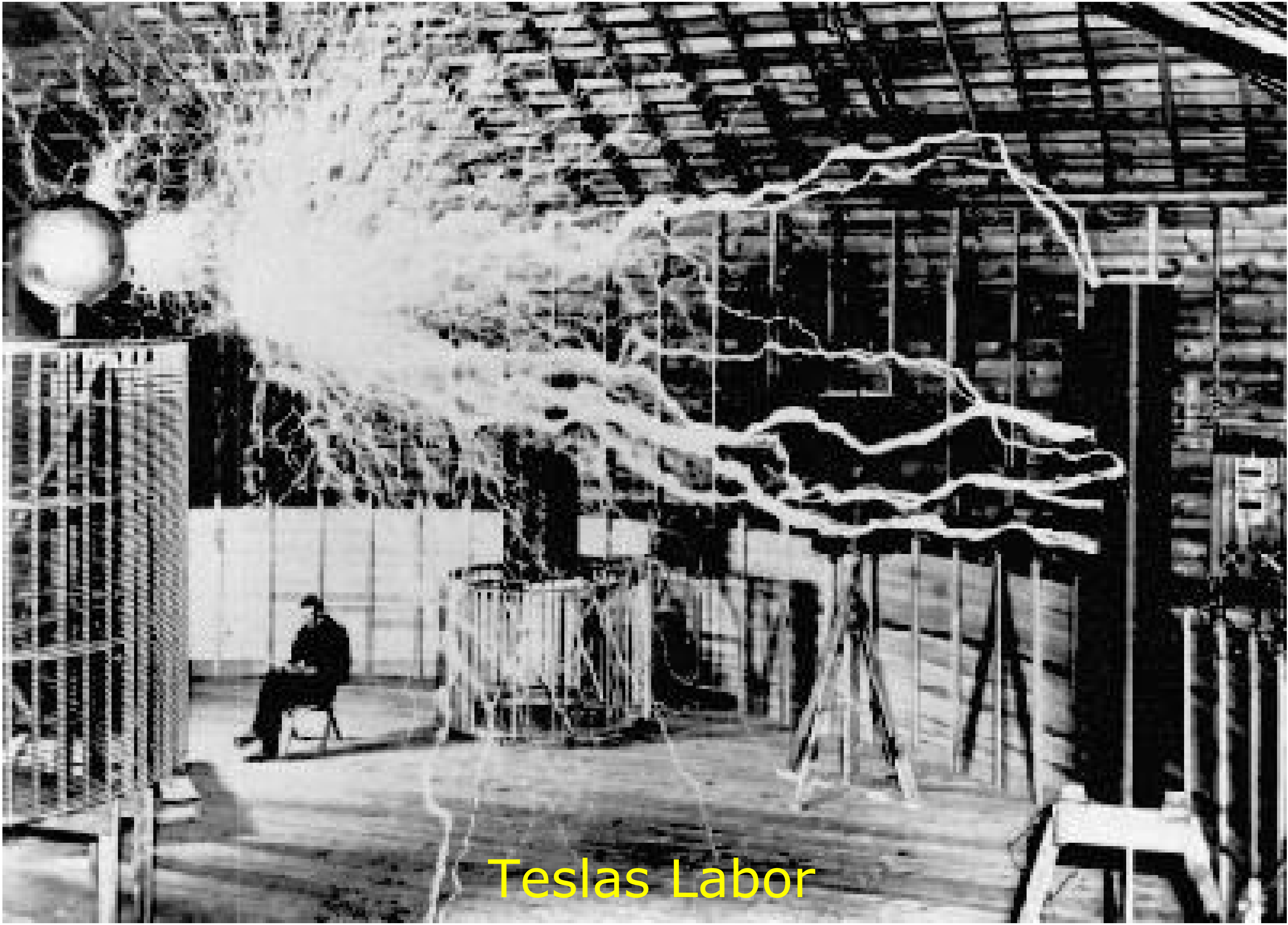
$$Q \approx \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\delta} \approx \sqrt{\frac{L}{C}}\,\frac{1}{R}$$

typ. Q : elektr 100

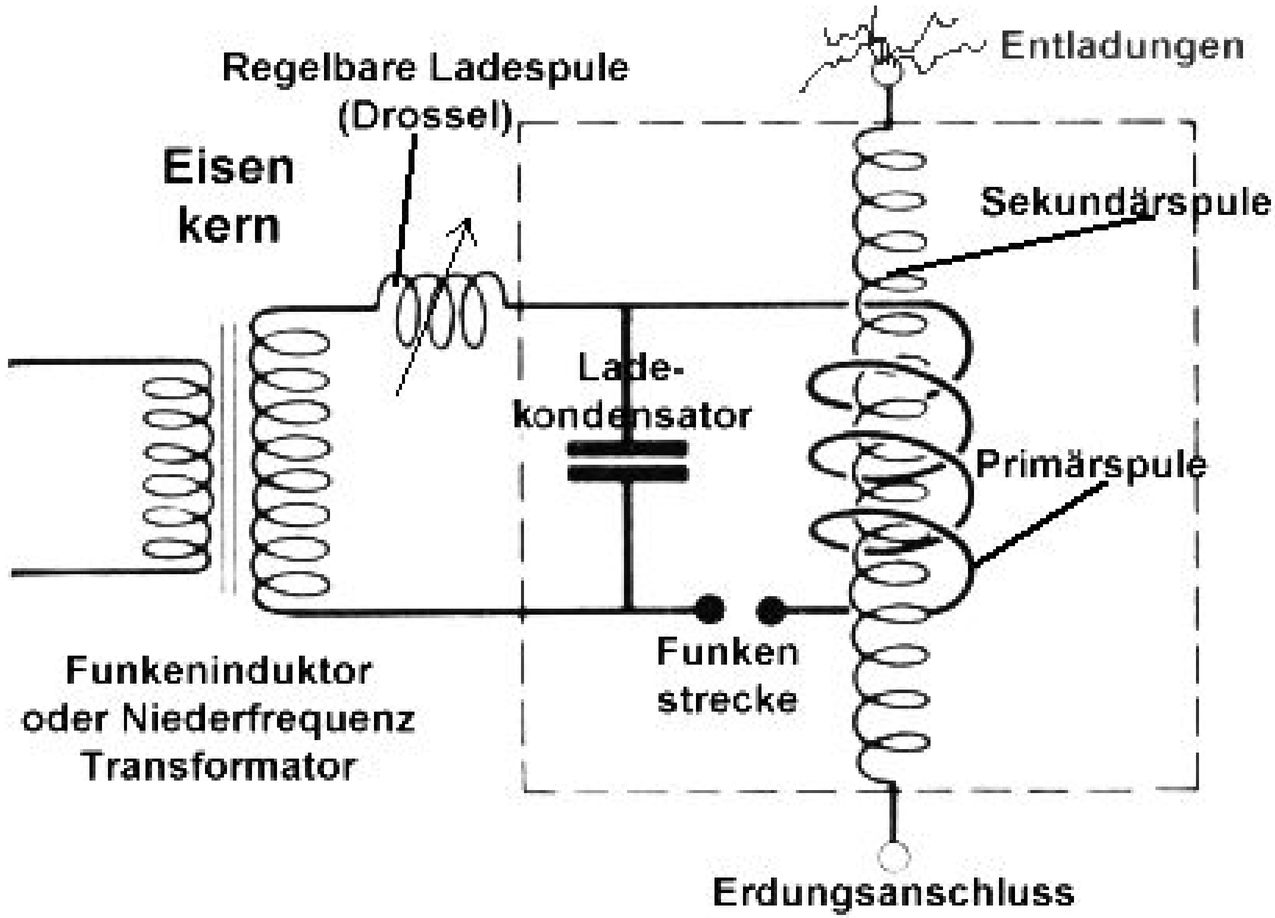
Pendeluhr 10^4

Quarz 10^6

Mößbauer 10^{15}



Teslas Labor



50 Hz Trafo erzeugt 10 kV
In einer Halbperiode: Kondensator (Leidener Flasche) aufladen
Bei Funken Sekundärseite kurzgeschlossen.
Schwingkreis Kondensator/ Primärspule des Teslatrafos 0,5 MHz
Sekundärspule des Teslatrafos in Resonanz, induktiv gekoppelt
(Tesla“trafo“: eigentlich Kopplung von Schwingkreisen)